Proof By Contradiction - $\sqrt{2}$ is not Rational

Proof that $\sqrt{2}$ is irrational, that it cannot be written $\frac{p}{q}$ where p and q are integers.

Suppose that $\sqrt{2}$ is rational, that it **can** be written as a fraction $\frac{p}{q}$ where p and q are integers with no common factors.

$$\left(\frac{p}{q}\right) = \sqrt{2}$$

square both sides

$$\left(\frac{p}{q}\right)^2 = 2$$

which implies

$$p^2 = 2q^2$$

The premise $p^2 = 2q^2$ tells us that p is even. Assuming p and q have no common factors, q must be odd. However the square of an even number is divisible by 4, which leads us to conclude that q is even. A contradiction.

Hence $\sqrt{2}$ cannot be written as a fraction $\frac{p}{q}$ where p and q are integers.