De Moivre's Theorem by Induction

De Moivre's Theorm

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Show true for $n = 1$

$(\cos\theta + i\sin\theta)^1 = \cos 1\theta + i\sin 1\theta$
Which is true.

Assume true for $n = k$
$(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$

Prove true for $n = k + 1$
$(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$

Proof

$(\cos\theta + i\sin\theta)^{k+1} = (\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)^1$

$= (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$

$= \cos k\theta\cos\theta + i\cos k\theta\sin\theta + i\cos\theta\sin k\theta + i^2\sin k\theta\sin\theta$

$= \boxed{\cos k\theta\cos\theta} + \boxed{i\cos k\theta\sin\theta} + \boxed{i\cos\theta\sin k\theta} + \boxed{(-1)\sin k\theta\sin\theta}$

$= \boxed{(\cos k\theta\cos\theta - \sin k\theta\sin\theta)} + \boxed{i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)}$

$= \cos(k\theta + \theta) + i\sin(k\theta + \theta)$

$= \cos(k+1)\theta + i\sin(k+1)\,\theta$

The proposition is true for $n = 1$. If the proposition is true for $n = k$, then it will be true for $n = k + 1$. Therefore by induction it is true for all $n \in \mathbb{N}$