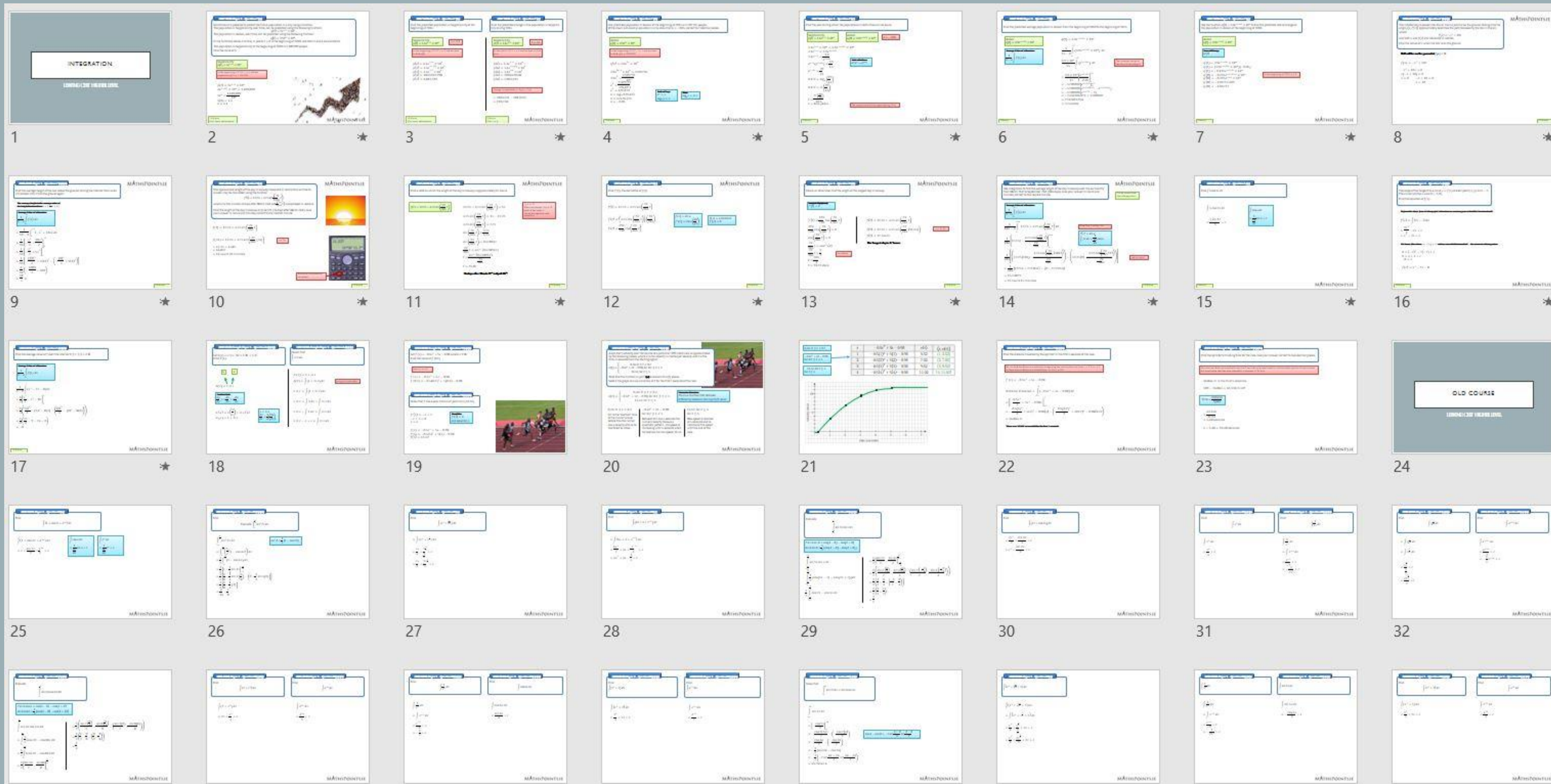
# INTEGRATION
## BASICS

Leaving Cert Higher level

# PREMIUM VERSION PREVIEW



The inflated ball is kicked into the air from a point $O$ on the ground. Taking $O$ as the origin, $(x, f(x))$ approximately describes the path followed by the ball in the air, where

$$f(x) = -x^2 + 10x$$

and both $x$ and $f(x)$ are measured in metres.

Find the values of $x$ when the ball is on the ground.

**Ball will be on the ground at $\boldsymbol{f(x) = 0}$**

$$f(x) = -x^2 + 10x$$

$$-x^2 + 10x = 0$$
$$x(-x + 10) = 0$$
$$x = 0 \qquad -x + 10 = 0$$
$$x = 10$$

10 Marks

Find the average height of the ball above the ground, during the interval from when it is kicked until it hits the ground again.

**The average height is the average value of the height function from $h = 0$ to $h = 10$**

**Average Value of a Function**

$$\frac{1}{b-a}\int_a^b f(x)\ dx$$

$$= \frac{1}{10-0}\int_0^{10} (-x^2 + 10x)\ dx$$

$$= \frac{1}{10}\left[-\frac{x^3}{3} + \frac{10x^2}{2}\right]_0^{10}$$

$$= \frac{1}{10}\left[-\frac{x^3}{3} + 5x^2\right]_0^{10}$$

$$= \frac{1}{10}\left[-\frac{(10)^3}{3} + 5(10)^2 - \left(-\frac{(0)^3}{3} + 5(0)^2\right)\right]$$

$$= \frac{1}{10}\left(-\frac{1000}{3} + 500\right)$$

$$= \frac{50}{3}\ \text{m}$$

10 Marks

## 2014 LCHL Sample Paper 1 – Question 6 (b) (i)

Let $h(x) = x \ln x$, for $x \in \mathbb{R},\ x > 0$.
Find $h'(x)$.

$u$ $v$

$$h(x) = x \ln x$$

**Product Rule**
$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$h'(x) = x\left(\frac{1}{x}\right) + \ln x\,(1)$$
$$h'(x) = 1 + \ln x$$

$$y = \ln u$$
$$\frac{dy}{dx} = \frac{1}{u}\left(\frac{du}{dx}\right)$$

## 2014 LCHL Sample Paper 1 – Question 6 (b) (i)

Hence, find
$$\int \ln x\, dx.$$

$$h'(x) = 1 + \ln x$$
$$h(x) = \int (1 + \ln x)\, dx$$

Integrate both sides.

$$x \ln x = \int (1 + \ln x)\, dx$$
$$x \ln x = \int 1\, dx + \int \ln x\, dx$$
$$x \ln x - \int 1\, dx = \int \ln x\, dx$$
$$x \ln x - x + c = \int \ln x\, dx$$

# OLD COURSE

Leaving Cert Higher level

## 2012 LCHL Paper 1 – Question 8 (a)

Find

$$\int (1 + \cos 2x + e^{3x})\, dx$$

$$\int (1 + \cos 2x + e^{3x})\, dx$$

$$= x + \frac{\sin 2x}{2} + \frac{e^{3x}}{3} + c$$

$$\int \cos u\, dx$$

$$= \frac{1}{\frac{du}{dx}} \sin u + c$$

$$\int e^{u}\, dx$$

$$= \frac{1}{\frac{du}{dx}} e^{u} + c$$

## 2012 LCHL Paper 1 – Question 8 (b) (ii)

Find

$$\text{Evaluate } \int_0^{\frac{\pi}{8}} \sin^2 2x \, dx$$

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

$$\int_0^{\frac{\pi}{8}} \sin^2 2x \, dx$$

$$= \int_0^{\frac{\pi}{8}} \left(\frac{1}{2}(1 - \cos 4x)\right) dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{8}} (1 - \cos 4x) \, dx$$

$$= \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{8}}$$

$$= \frac{1}{2}\left[\frac{\pi}{8} - \frac{1}{4}\sin 4\left(\frac{\pi}{8}\right) - \left(0 - \frac{1}{4}\sin 4(0)\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{8} - \frac{1}{4}(1)\right]$$

$$= \frac{\pi}{16} - \frac{1}{8}$$

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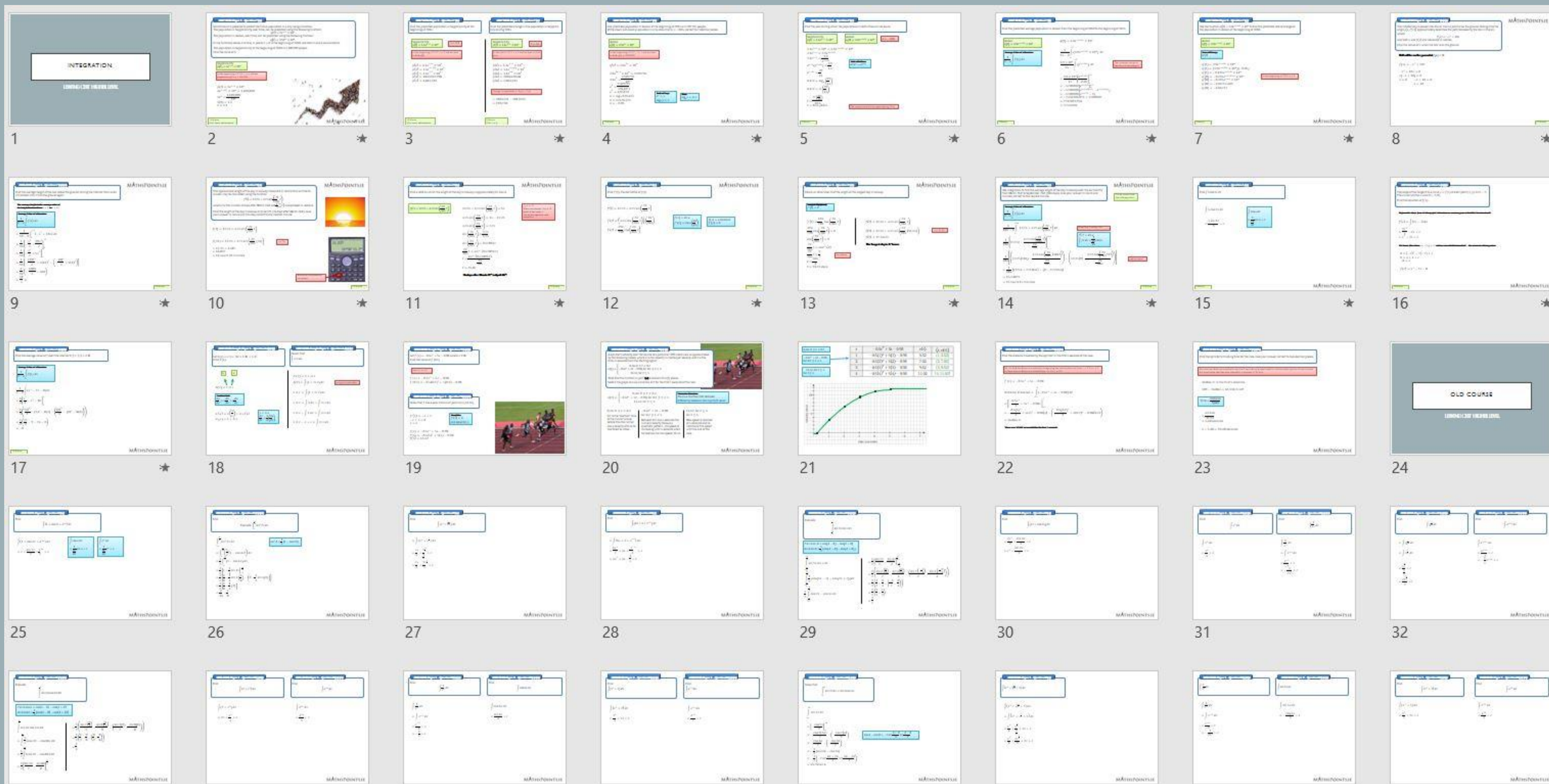


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