MATHSPOINTS.IE
JUNIOR & LEAVING CERT

ALGEBRA – LOGS AND INDICES
(NON REAL WORLD)

LEAVING CERT HIGHER LEVEL

Algebra – Logs and Indices

LCHL New Course

- 2016 Paper 1 – Q4 (b)
- 2014S Paper 1 – Q2 (b)

LCOL New Course

- 2014S Paper 1 – Q1 (a)
- 2014S Paper 1 – Q1 (c)
- 2014S Paper 1 – Q1 (d)
- 2013 Paper 1 – Q3 (c)

JCHL New Course

- 2017 Paper 1 – Q10
- 2016 Paper 1 – Q9 (a)
- 2016 Paper 1 – Q9 (b)

LCHL Old Course

- 2012 Paper 1 – Q5 (a)
- 2011 Paper 1 – Q5 (b) (i)
- 2011 Paper 1 – Q5 (b) (ii)
- 2010 Paper 1 – Q5 (a)
- 2009 Paper 1 – Q5 (c)
- 2008 Paper 1 – Q5 (b) (i)
- 2008 Paper 1 – Q5 (b) (ii)
- 2006 Paper 1 – Q2 (c)
- 2006 Paper 1 – Q5 (c)
- 2005 Paper 1 – Q1 (b) (i)
- 2005 Paper 1 – Q5 (c)
- 2004 Paper 1 – Q1 (b) (ii)
- 2004 Paper 1 – Q5 (b) (ii)
- 2003 Paper 1 – Q2 (c)
- 2002 Paper 1 – Q5 (a)
- 2001 Paper 1 – Q5 (b) (i)
- 2000 Paper 1 – Q5 (c) (i)
- 2000 Paper 1 – Q5 (c) (ii)

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ALGEBRA – LOGS AND INDICES
(NON REAL WORLD)

Leaving Cert Higher Level

Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

$\log_a \frac{8}{3}$

$\log_a \frac{8}{3}$
$= \log_a 8 - \log_a 3$
$= \log_a (2)^3 - \log_a 3$
$= 3\log_a 2 - \log_a 3$
$= 3p - q$

Rules of Logs

$\log_a m + \log_a n = \log_a mn$

$\log_a m - \log_a n = \log_a \frac{m}{n}$

$n \log_a m = \log_a m^n$

$\log_n m = \frac{\log_a m}{\log_a n}$

(ii)

$\log_a \frac{9a^2}{16}$

$\log_a \frac{9a^2}{16}$
$= \log_a 9a^2 - \log_a 16$
$= \log_a (3a)^2 - \log_a (2)^4$
$= 2\log_a 3a - 4\log_a 2$
$= 2(\log_a 3 + \log_a a) - 4\log_a 2$
$= 2(\log_a 3 + 1) - 4\log_a 2$
$= 2\log_a 3 + 2 - 4\log_a 2$
$= 2q + 2 - 4p$
$= -4p + 2q + 2$

Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c(cx)$ in terms of p.

$$\log_c \sqrt{x} + \log_c cx$$
$$= \log_c x^{\frac{1}{2}} + \log_c c + \log_c x$$
$$= \frac{1}{2}\log_c x + \log_c c + \log_c x$$
$$= \frac{1}{2}p + 1 + p$$
$$= \frac{3}{2}p + 1$$

Note:
$\sqrt{x} = x^{\frac{1}{2}}$

Note:
$\log_a a = 1$

Rules of Logs

$$\log_a m + \log_a n = \log_a mn$$
$$\log_a m - \log_a n = \log_a \frac{m}{n}$$
$$n \log_a m = \log_a m^n$$
$$\log_n m = \frac{\log_a m}{\log_a n}$$

ALGEBRA – INDICES

Leaving Cert Ordinary Level

Write 6^{-2} and $81^{\frac{1}{2}}$ without using indices.

$$6^{-2} = \frac{1}{6^2}$$
$$= \frac{1}{36}$$

$$81^{\frac{1}{2}} = \sqrt{81}$$
$$= 9$$

Rules of Indices

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

Show that $\frac{(a\sqrt{a})^3}{a^4}$ simplifies to $\sqrt{a}$.

$$\frac{(a\sqrt{a})^3}{a^4}$$
$$= \frac{\left(a\left(a^{\frac{1}{2}}\right)\right)^3}{a^4}$$
$$= \frac{\left(a^{\frac{3}{2}}\right)^3}{a^4}$$
$$= \frac{a^{\frac{9}{2}}}{a^4}$$
$$= a^{\frac{1}{2}}$$

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Solve the equation $49^x = 7^{2+x}$ and verify your answer.

$49^x = 7^{2+x}$
$(7^2)^x = 7^{2+x}$
$7^{2x} = 7^{2+x}$

$2x = 2 + x$
$2x - x = 2$
$x = 2$

$49^x = 7^{2+x}$
$49^2 = 7^{2+2}$
$49^2 = 7^4$
$2401 = 2401$

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Solve the equation $27^{2x} = 3^{x+10}$.

$27^{2x} = 3^{x+10}$

$(3^3)^{2x} = 3^{x+10}$

$3^{6x} = 3^{x+10}$

$6x = x + 10$

$5x = 10$

$x = 2$

Rules of Indices

$a^p a^q = a^{p+q}$

$\dfrac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \dfrac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$

ALGEBRA – INDICES

Junior Cert Higher Level

Write each of the following in the form 2^n, where $n \in \mathbb{Q}$.

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Rules Applied

(a)

$2^3 \times 2^5 \times 2^{10}$

$$2^3 \times 2^5 \times 2^{10}$$
$$= 2^{18}$$

Rule applied: $a^p a^q = a^{p+q}$

(b)

8^{25}

$$8^{25}$$
$$= (2^3)^{25}$$
$$= 2^{75}$$

Rules applied: $8 = 2^3$; $(a^p)^q = a^{pq}$

(c)

$\sqrt{8}$

$$\sqrt{8}$$
$$= 8^{\frac{1}{2}}$$
$$= (2^3)^{\frac{1}{2}}$$
$$= 2^{\frac{3}{2}}$$

Rules applied: $\sqrt[q]{a} = a^{\frac{1}{q}}$; $8 = 2^3$; $(a^p)^q = a^{pq}$

Write each of the following numbers in the form 3^k, where $k \in \mathbb{Q}$.

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

(i)

9

$9 = 3^2$

(ii)

1

$1 = 3^0$

(iii)

$\sqrt{27}$

$\sqrt{27}$

$= (27)^{\frac{1}{2}}$

$= (3^3)^{\frac{1}{2}}$

$= 3^{\frac{3}{2}}$

(iv)

$\frac{1}{\sqrt[3]{3}}$

$\frac{1}{\sqrt[3]{3}}$

$= \frac{1}{3^{\frac{1}{3}}}$

$= 3^{-\frac{1}{3}}$

Write $(-2n)^4$, in the form $a\ n^b$ where $a, b \in \mathbb{Z}$.

$$(-2n)^4$$
$$= (-2)^4\ (n)^4$$
$$= 16n^4$$

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

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LEAVING CERT HIGHER LEVEL

(OLD COURSE)

Solve for $x \in \mathbb{R}$:

$$\log_4(2x+6) - \log_4(x-1) = 1$$

Rules of Logs

$\log_a m + \log_a n = \log_a mn$

$\log_a m - \log_a n = \log_a \frac{m}{n}$

$n \log_a m = \log_a m^n$

$\log_n m = \frac{\log_a m}{\log_a n}$

$$\log_4(2x+6) - \log_4(x-1) = 1$$

$$\log_4\left(\frac{2x+6}{x-1}\right) = 1$$

$$4^1 = \frac{2x+6}{x-1}$$

$$4x - 4 = 2x + 6$$

$$4x - 2x = 6 + 4$$

$$2x = 10$$

$$x = 5$$

Solve the equation:

$$\log_2 x - \log_2(x-1) = 4\log_4 2$$

$$\log_2 x - \log_2(x-1) = 4\log_4 2$$
$$\log_2\left(\frac{x}{x-1}\right) = 2$$
$$2^2 = \frac{x}{x-1}$$
$$4 = \frac{x}{x-1}$$
$$4x - 4 = x$$
$$4x - x = 4$$
$$3x = 4$$
$$x = \frac{4}{3}$$

Solve the equation:

$$3^{2x+1} - 17(3^x) - 6 = 0$$

Give your answer correct to two decimal places.

$3^{2x+1} - 17(3^x) - 6 = 0$
$3^{2x}.3^1 - 17(3^x) - 6 = 0$
$(3^x)^2.3^1 - 17(3^x) - 6 = 0$
$3(3^x)^2 - 17(3^x) - 6 = 0$

$3(y)^2 - 17(y) - 6 = 0$

For simplicity let $y = 3^x$

$3y^2 - 17y - 6 = 0$
$(3y + 1)(y - 6) = 0$

$3y + 1 = 0$
$3y = -1$
$y = -\frac{1}{3}$

$y - 6 = 0$
$y = 6$

$3^x = -\frac{1}{3}$
Not a Solution

$3^x = 6$
$x = \log_3 6$
$x = 1.63$

Solve the equation:

$$\log_2(x+6) - \log_2(x+2) = 1$$

$$\log_2(x+6) - \log_2(x+2) = 1$$

$$\log_2\left(\frac{x+6}{x+2}\right) = 1$$

$$2^1 = \frac{x+6}{x+2}$$

$$2x + 4 = x + 6$$

$$2x - x = 6 - 4$$

$$x = 2$$

Solve the simultaneous equations

$$\log_3 x + \log_3 y = 2$$
$$\log_3(2y - 3) - 2\log_9 x = 1$$

$$\log_3 x + \log_3 y = 2$$
$$\log_3(xy) = 2$$

$$3^2 = xy$$
$$9 = xy$$

$$9 = \left(\frac{2y-3}{3}\right)y$$

$$27 = 2y^2 - 3y$$
$$2y^2 - 3y - 27 = 0$$
$$(2y - 9)(y + 3) = 0$$
$$2y - 9 = 0 \qquad y + 3 = 0$$
$$2y = 9 \qquad y = -3$$
$$y = \frac{9}{2}$$

$y = -3$ not a solution as $y > 0$

$$\log_3(2y - 3) - 2\log_9 x = 1$$
$$\log_3(2y - 3) - \log_9 x^2 = 1$$
$$\log_3(2y - 3) - \frac{\log_3 x^2}{\log_3 9} = 1$$
$$\log_3(2y - 3) - \frac{\log_3 x^2}{2} = 1$$
$$\log_3(2y - 3) - \frac{1}{2}\log_3 x^2 = 1$$
$$\log_3(2y - 3) - \log_3 x = 1$$
$$\log_3\left(\frac{2y-3}{x}\right) = 1$$
$$3^1 = \frac{2y-3}{x}$$
$$x = \frac{2y-3}{3}$$

$$x = \frac{2y-3}{3}$$
$$x = \frac{2\left(\frac{9}{2}\right) - 3}{3}$$
$$x = 2$$

Solve the equation

$$2^{x^2} = 8^{2x+9}$$

$$2^{x^2} = 8^{2x+9}$$
$$2^{x^2} = (2^3)^{2x+9}$$
$$2^{x^2} = 2^{6x+27}$$
$$x^2 = 6x + 27$$
$$x^2 - 6x - 27 = 0$$
$$(x-9)(x+3) = 0$$

$x - 9 = 0$	$x + 3 = 0$
$x = 9$	$x = -3$

Solve the equation

$$\log_e(2x+3) + \log_e(x-2) = 2\log_e(x+4)$$

$$\log_e(2x+3) + \log_e(x-2) = 2\log_e(x+4)$$
$$\log_e\big((2x+3)(x-2)\big) = \log_e(x+4)^2$$
$$(2x+3)(x-2) = (x+4)^2$$
$$2x^2 + 3x - 4x - 6 = x^2 + 8x + 16$$
$$x^2 - 9x - 22 = 0$$
$$(x-11)(x+2) = 0$$
$$x = 11 \qquad x = -2$$

$x = 11$ is the only valid solution.

$f(x) = 1 - b^{2x}$ and $g(x) = b^{1+2x}$, where b is a positive number.

Find, in terms of b, the value of x for which $f(x) = g(x)$.

$$f(x) = g(x)$$

$$1 - b^{2x} = b^{1+2x}$$

$$1 - b^{2x} = b^1 b^{2x}$$

$$1 = b^{2x}(b+1)$$

$$\frac{1}{b+1} = b^{2x}$$

$$b^{2x} = \frac{1}{b+1}$$

$$\log_b b^{2x} = \log_b \frac{1}{b+1}$$

$$2x\log_b b = \log_b \left(\frac{b+1}{1}\right)^{-1}$$

$$2x\log_b b = -1 \log_b (b+1)$$

$$2x = -1 \log_b (b+1)$$

$$x = -\frac{1}{2}\log_b (b+1)$$

$$x = -\log_b (b+1)^{\frac{1}{2}}$$

$$x = -\log_b \sqrt{b+1}$$

(i) Given two real numbers a and b, where $a > 1$ and $b > 1$, prove that

$$\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2$$

(ii) Under what condition is

$$\frac{1}{\log_b a} + \frac{1}{\log_a b} = 2$$

(i)

$$\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2$$

$$\log_a b + \frac{1}{\log_a b} \geq 2$$

$$(\log_a b)^2 + 1 \geq 2\log_a b$$

$$(\log_a b)^2 - 2\log_a b + 1 \geq 0$$

$$(\log_a b - 1)(\log_a b - 1) \geq 0$$

$$(\log_a b - 1)^2 \geq 0$$

True

(i)

$$\frac{1}{\log_b a} + \frac{1}{\log_a b} = 2$$

True when

$$(\log_a b - 1)^2 = 0$$

$$\log_a b - 1 = 0$$

$$\log_a b = 1$$

$$a = b$$

Logs Rule

$$\frac{1}{\log_b a} = \log_a b$$

Logs Rule

$a > 1, b > 1$ then

$\log_a b > 0$

Express $2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}}$ in the form $2^{\frac{p}{q}}$, where $p, q \in Z$.

$2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}}$	**Given**
$= 4\left(2^{\frac{1}{4}}\right)$	**There are four $2^{\frac{1}{4}}$'s therefore $2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} = 4\left(2^{\frac{1}{4}}\right)$**
$= 2^2\left(2^{\frac{1}{4}}\right)$	**Write 4 as 2^2**
$= 2^{2\frac{1}{4}}$	**When multiplying numbers of the same base we add the powers**
$= 2^{\frac{9}{2}}$	**Turn into top heavy fraction to leave in the form $2^{\frac{p}{q}}$**

(i) Show that $\frac{1}{\log_a b} = \log_b a$, where $a, b > 0$ and $a, b \neq 1$.

(ii) Show that $\frac{1}{\log_2 c} + \frac{1}{\log_3 c} + \frac{1}{\log_4 c} + \ldots + \frac{1}{\log_r c} = \frac{1}{\log_{r!} c}$, where $c > 0$, $c \neq 1$.

Show that

$$\frac{3}{1+x^p}+\frac{3}{1+x^{-p}}$$

simplifies to a constant.

$$\frac{3}{1+x^p}+\frac{3}{1+x^{-p}}$$
$$=\frac{3}{1+x^p}+\frac{3}{1+\frac{1}{x^p}}$$
$$=\frac{3}{1+x^p}+\frac{3}{\frac{x^p+1}{x^p}}$$
$$=\frac{3}{1+x^p}+\frac{3x^p}{x^p+1}$$
$$=\frac{3+3x^p}{1+x^p}$$
$$=\frac{3(1+x^p)}{1+x^p}$$
$$=3$$

Solve

$$\log_4 x - \log_4(x-2) = \frac{1}{2}.$$

Rules of Logs

$\log_a m + \log_a n = \log_a mn$

$\log_a m - \log_a n = \log_a \frac{m}{n}$

$n \log_a m = \log_a m^n$

$\log_n m = \frac{\log_a m}{\log_a n}$

$$\log_4 x - \log_4(x-2) = \frac{1}{2}$$
$$\log_4\left(\frac{x}{x-2}\right) = \frac{1}{2}$$
$$4^{\frac{1}{2}} = \frac{x}{x-2}$$
$$2 = \frac{x}{x-2}$$
$$2x - 4 = x$$
$$x = 4$$

Solve for y:

$$2^{2y+1} - 5(2^y) + 2 = 0.$$

$$2^{2y+1} - 5(2^y) + 2 = 0$$
$$2^{2y}.2^1 - 5(2^y) + 2 = 0$$
$$2(2^y)^2 - 5(2^y) + 2 = 0$$

Let $x = 2^y$

$$2x^2 - 5x + 2 = 0$$
$$(2x - 1)(x - 2) = 0$$

$2x - 1 = 0$ | $x - 2 = 0$

$2x = 1$ | $x = 2$

$x = \frac{1}{2}$

$$2^y = \frac{1}{2}$$
$$y = \log_2 \frac{1}{2}$$
$$y = -1$$

$$2^y = 2$$
$$y = 1$$

Find the value of x:

$\frac{8}{2^x} = 32$

$$\frac{8}{2^x} = 32$$
$$\frac{8}{32} = 2^x$$
$$\log_2 \frac{8}{32} = x$$
$$-2 = x$$

OR

$$\frac{8}{2^x} = 32$$
$$\frac{2^3}{2^x} = 2^5$$
$$2^{3-x} = 2^5$$
$$3 - x = 5$$
$$-2 = x$$

(ii)

Find the value of x:

$\log_9 x = \frac{3}{2}$

$$\log_9 x = \frac{3}{2}$$
$$9^{\frac{3}{2}} = x$$
$$27 = x$$

Solve $\log_6(x+5) = 2 - \log_6 x$ for $x > 0$.

Rules of Logs

$\log_a m + \log_a n = \log_a mn$

$\log_a m - \log_a n = \log_a \frac{m}{n}$

$n \log_a m = \log_a m^n$

$\log_n m = \frac{\log_a m}{\log_a n}$

$\log_6(x+5) = 2 - \log_6 x$
$\log_6(x+5) + \log_6 x = 2$
$\log_6 x(x+5) = 2$
$x(x+5) = 6^2$
$x^2 + 5x = 36$
$x^2 + 5x - 36 = 0$
$(x+9)(x-4) = 0$
$x = -9 \quad x = 4$

$x > 0$
$\therefore x = 4$

Solve for x

$$2\log_9 x = \frac{1}{2} + \log_9(5x+18)\,, \qquad x > 0$$

Rules of Logs

$\log_a m + \log_a n = \log_a mn$

$\log_a m - \log_a n = \log_a \frac{m}{n}$

$n \log_a m = \log_a m^n$

$\log_n m = \frac{\log_a m}{\log_a n}$

$$2\log_9 x = \frac{1}{2} + \log_9(5x+18)$$
$$\log_9 x^2 - \log_9(5x+18) = \frac{1}{2}$$
$$\log_9\left(\frac{x^2}{5x+18}\right) = \frac{1}{2}$$
$$\frac{x^2}{5x+18} = 9^{\frac{1}{2}}$$
$$\frac{x^2}{5x+18} = 3$$
$$x^2 = 15x + 54$$
$$x^2 - 15x - 54 = 0$$
$$(x-18)(x+3) = 0$$

$x - 18 = 0$ $\qquad$ $x + 3 = 0$

$x = 9$ $\qquad$ $x = -3$

Invalid solution

Solve for x

$$3e^x - 7 + 2e^{-x} = 0.$$

$$3e^x - 7 + 2e^{-x} = 0$$

$$3e^x - 7 + \frac{2}{e^x} = 0$$

Let $y = e^x$

$$3y - 7 + \frac{2}{y} = 0$$
$$3y^2 - 7y + 2 = 0$$
$$(3y - 1)(y - 2) = 0$$

$$3y - 1 = 0 \qquad y - 2 = 0$$
$$3y = 1 \qquad y = 2$$
$$y = \frac{1}{3}$$

$$y = e^x$$
$$e^x = \frac{1}{3}$$
$$x = \ln\frac{1}{3}$$

$$y = e^x$$
$$e^x = 2$$
$$x = \ln 2$$