

MATHSPOINTS.IE
JUNIOR & LEAVING CERT

ALGEBRA REFERENCE SHEET

LEAVING CERT ORDINARY LEVEL

Substitution

Calculate the value of $p^2 - 3rq$
 For $p = 3, q = -4$ and $r = 7$

Sub in values for p, q and r
 $(3)^2 - 3(7)(-4)$
 $= 9$

The amount of wheat yielded (Y) depends on the amount of nitrogen spread (N) in the following way:
 $Y = 7000 + 32N - 0.1N^2$
 How much wheat will he get per hectare if he uses 100 kg of nitrogen per hectare?
 $Y = 7000 + 32(100) - 0.1(100)^2$
 $Y = 9200$

Simplify

Simplify $3(4 - 5x) - 2(5 - 6x)$.

$$3(4 - 5x) - 2(5 - 6x)$$

$$= 12 - 15x - 10 + 12x$$

$$= 2 - 3x$$

Express as a Single Fraction

Write as a single fraction
 $\frac{3x - 2}{4} + \frac{2x}{3} - \frac{5x - 1}{12}$

Find a common denominator.

$$= \frac{3(3x - 2) + 4(2x) - 1(5x - 1)}{12}$$

$$= \frac{9x - 6 + 8x - 5x + 1}{12}$$

$$= \frac{17x - 5}{12}$$

Multiplying Brackets (the opposite of Factorising)

$$(3x + 2)(4x - 3)$$

$$= 3x(4x - 3) + 2(4x - 3)$$

$$= 12x^2 - 9x + 8x - 6$$

$$= 12x^2 - x - 6$$

Manipulate Formulae (Rearranging)

This is taking a formula and changing its subject (the letter by itself) through rules of maths. (Multiplication, factorising etc)

$$M = \frac{1}{S + P}$$

Rearrange the formula to make P its subject.

$$M(S + P) = 1$$

$$MS + MP = 1$$

$$MP = 1 - MS$$

$$P = \frac{1 - MS}{M}$$

Rearrange the formula $P = \frac{M}{h^3}$ to give a formula which will give the height, h of a person in terms of their weight, M and Ponderal index, P .

$$P = \frac{M}{h^3}$$

$$Ph^3 = M$$

$$h^3 = \frac{M}{P}$$

$$h = \sqrt[3]{\frac{M}{P}}$$

Factorising – 4 types

Highest Common Factor

$$6x^2 - 15xy$$

$$= 3x(2x - 5y)$$

Difference of Squares

$$4x^2 - 81$$

$$= (2x + 9)(2x - 9)$$

Quadratics

$$6x^2 - 5x - 21$$

$$= (3x - 7)(2x + 3)$$

Grouping

$$9a^2 - 6ab + 12ac - 8bc$$

$$= 3a(3a - 2b) + 4c(3a - 2b)$$

$$= (3a + 4c)(3a - 2b)$$

Combinations

Simplify

$$\frac{8x^2 - 12x}{4x^2 - 12x + 9}$$

$$= \frac{4x(2x - 3)}{(2x - 3)(2x - 3)}$$

$$= \frac{4x}{2x - 3}$$

Algebra Division

Divide $2x^3 + x^2 - 13x + 6$ by $x + 3$

$$\begin{array}{r}
 2x^2 - 5x + 2 \\
 x + 3 \overline{) 2x^3 + x^2 - 13x + 6} \\
 \underline{-2x^3 + 6x^2} \\
 5x^2 - 13x + 6 \\
 \underline{+5x^2 - 15x} \\
 2x + 6 \\
 \underline{-2x + 6} \\
 0
 \end{array}$$

$g(x)$ is a function and $(2 - 3x) \times g(x) = 15x^2 - 22x + 8$, for all $x \in \mathbb{R}$. Find $g(x)$.
 The opposite of multiplication is division so to find $g(x)$ divide.

$$\begin{array}{r}
 -5x + 4 \\
 -3x + 2 \overline{) 15x^2 - 22x + 8} \\
 \underline{-15x^2 + 10x} \\
 -12x + 8 \\
 \underline{+12x - 8} \\
 0
 \end{array}$$

Simple Equation

Multiply to get rid of brackets, x 's to one side, numbers to the other ...

Solve for x

$$3(2x - 1) = 4x$$

$$6x - 3 = 4x$$

$$6x - 4x = 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

Solve for x :

$$2(4 - 3x) + 12 = 7x - 5(2x - 7)$$

$$2(4 - 3x) + 12 = 7x - 5(2x - 7)$$

$$8 - 6x + 12 = 7x - 10x + 35$$

$$8 - 35 + 12 = 7x - 10x + 6x$$

$$-15 = 3x$$

$$\frac{-15}{3} = x$$

$$-5 = x$$

Solve the equation

$$\frac{1}{2}(7x - 2) + 5 = 2x + 7$$

$$\frac{1}{2}(7x - 2) + 5 = 2x + 7$$

$$7x - 2 + 10 = 4x + 14$$

$$7x - 4x = 14 - 10 + 2$$

$$3x = 6$$

$$x = 2$$

Inequalities

Treat this like an equation with x 's to one side and numbers to the other.

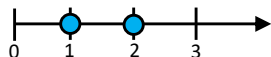
$x \in N$, $x \in Z$ means dots.

$x \in R$ means shading

$$5x + 1 \leq 4x + 3, x \in N$$

$$5x - 4x \leq 3 - 1$$

$$x \leq 2$$



Algebra - Leaving Cert Ordinary Level Reference Sheet

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Compound Inequalities

$$-2 \leq 5x + 3 < 18, x \in R$$

Split into two inequalities and solve as before.

$$-2 \leq 5x + 3 \quad 5x + 3 < 18$$

$$-2 - 3 \leq 5x \quad 5x < 18 - 3$$

$$-5 \leq 5x \quad 5x < 15$$

$$-1 \leq x \quad x < 3$$



Forming a Quadratic Equation

$$x^2 - (\text{sum of the roots})x + (\text{product of the roots}) = 0$$

Form the equation with roots $x = 4$ and $x = 7$

$$x^2 - (4 + 7)x + (4)(7) = 0$$

$$x^2 - 11x + 28 = 0$$

Note - 'roots' are values for x that satisfy the equation.

Solving Quadratics

Solve the equation

$$x^2 - x - 6 = 0$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \quad x + 2 = 0$$

$$x = 3 \quad x = -2$$

$$f(x) = 2x^2 - 4x - 6$$

Solve for $f(x) = 0$

factorise $(2x - 6)(x + 1) = 0$

$$2x - 6 = 0 \quad x + 1 = 0$$

$$2x = 6 \quad x = -1$$

$$x = 3$$

or - b formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{16 + 48}}{4}$$

$$x = \frac{4 \pm \sqrt{64}}{4}$$

$$x = \frac{4 \pm 8}{4}$$

$$x = \frac{4 + 8}{4} \quad \& \quad \frac{4 - 8}{4}$$

$$x = 3 \quad \& \quad -1$$

Simultaneous Equations - 2 unknowns (linear)

$$\begin{aligned} \text{Solve} \quad 4x + 16y &= 20 \\ 2x - 3y &= -1 \end{aligned}$$

Multiply one or both lines to make co-efficients of one of the variables the same. Cancel down and solve.

$$\begin{array}{r} 4x + 16y = 20 \\ 2x - 3y = -1 \quad \times -2 \\ \hline 4x + 16y = 20 \\ -4x + 6y = 2 \\ \hline 22y = 22 \\ y = 1 \end{array}$$

$$\begin{aligned} 4x + 16y &= 20 \\ 4x + 16(1) &= 20 \\ 4x + 16 &= 20 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

Also occurs in co-ordinate geometry to find where lines intersect.

Simultaneous Equations - 2 unknowns (1 linear & 1 non-linear)

Solve the simultaneous equations:

$$\begin{aligned} x + y &= 7 \\ x^2 + y^2 &= 25 \end{aligned}$$

Take the linear expression and express one variable in terms of the other. Sub this into the non-linear and solve.

$$\begin{aligned} x + y &= 7 \\ x &= 7 - y \end{aligned}$$

$$\begin{aligned} x^2 + y^2 &= 25 \\ (7 - y)^2 + y^2 &= 25 \\ (7 - y)(7 - y) + y^2 &= 25 \\ 7(7 - y) - y(7 - y) + y^2 &= 25 \\ 49 - 7y - 7y + y^2 + y^2 &= 25 \\ 2y^2 - 14y + 24 &= 0 \\ y^2 - 7y + 12 &= 0 \\ (y - 4)(y - 3) &= 0 \\ y = 4 \quad y = 3 \end{aligned}$$

$$\begin{aligned} x &= 7 - y \\ @y &= 4 \\ x &= 7 - 4 \\ x &= 3 \end{aligned}$$

$$(3, 4)$$

$$\begin{aligned} @y &= 3 \\ x &= 7 - 3 \\ x &= 4 \end{aligned}$$

$$(4, 3)$$

Also occurs in co-ordinate geometry to find where a line intersects a circle.

Laws of Indices

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^n} = \frac{1}{a^n} \quad \left| \quad a^{\frac{m}{n}} = \sqrt[n]{a^m} \right.$$

$$a^0 = 1$$

Properties of Surds

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a}\sqrt{a} = a$$

Indices Equation

Solve the equation
 $27^{2x} = 3^{x+10}$

$$27^{2x} = 3^{x+10}$$

$$(3^3)^{2x} = 3^{x+10}$$

$$3^{6x} = 3^{x+10}$$

$$6x = x + 10$$

$$5x = 10$$

$$x = 2$$

Solve for x
 $8^x = 4^{2x+1}$

$$\frac{(2^3)^x}{2^2} = (2^2)^{2x+1}$$

$$\frac{2^{3x}}{2^2} = 2^{4x+2}$$

$$2^{3x-\frac{1}{2}} = 2^{4x+2}$$

$$3x - \frac{1}{2} = 4x + 2$$

$$2 - \frac{1}{2} = 4x - 3x$$

$$1\frac{1}{2} = x$$

Surds Equation

Solve for x
 $(x + \sqrt{x})(x - \sqrt{x}) = 6$

$$x(x - \sqrt{x}) + \sqrt{x}(x - \sqrt{x}) = 6$$

$$x^2 - x\sqrt{x} + x\sqrt{x} - \sqrt{x}^2 = 6$$

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3 \quad x = -2$$

not a solution

Must check (verify) the answers.

Verify that $3 + \sqrt{2}$ is a root (solution) of the equation $x^2 - 6x + 7 = 0$.

$$x^2 - 6x + 7 = 0$$

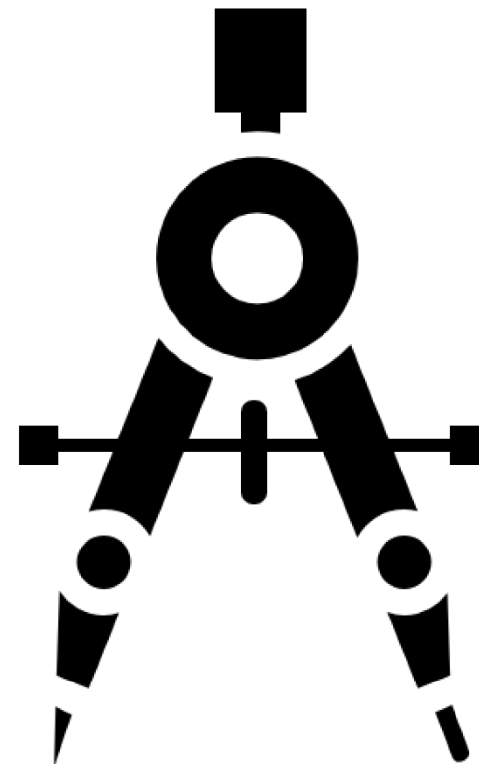
$$(3 + \sqrt{2})^2 - 6(3 + \sqrt{2}) + 7 = 0$$

$$3(3 + \sqrt{2}) + \sqrt{2}(3 + \sqrt{2}) - 6(3 + \sqrt{2}) + 7 = 0$$

$$9 + 3\sqrt{2} + 3\sqrt{2} + 2 - 18 - 6\sqrt{2} + 7 = 0$$

$$6\sqrt{2} - 6\sqrt{2} + 18 - 18 = 0$$

True



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Equations Using Fractions

Solve the equation

$$\frac{2}{3x-4} - \frac{1}{2x+1} = \frac{1}{2}$$

and give your answers correct to one decimal place.

$$\frac{2(2)(2x+1) - 1(2)(3x-4) = 1(3x-4)(2x+1)}{2(3x-4)(2x+1)}$$

$$\begin{aligned} 8x + 4 - 6x + 8 &= 6x^2 - 8x + 3x - 4 \\ 0 &= 6x^2 - 8x - 8x + 6x + 3x - 8 - 4 - 4 \\ 6x^2 - 7x - 16 &= 0 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-16)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{433}}{12}$$

$$x = \frac{7 + \sqrt{433}}{12} \text{ and } x = \frac{7 - \sqrt{433}}{12}$$

$$x = 2.3 \text{ \& } -1.2$$

Writing Expressions/ Solving Problems

The length of a rectangle is 5 times its width. The perimeter of the rectangle is 120m.

$$\begin{aligned} \text{width} &= x & \text{length} &= 5x \\ x + x + 5x + 5x &= 120 \\ x &= 10 \end{aligned}$$

x is a real number.

One new number is formed by increasing x by 1.

A second new number is formed by decreasing x by 2.

The product of the 2 new numbers is 1

Find x .

$$\begin{aligned} (x+1)(x-2) &= 1 \\ x(x-2) + 1(x-2) &= 1 \\ x^2 - 2x + x - 2 - 1 &= 0 \\ x^2 - x - 3 &= 0 \end{aligned}$$

Use $-b$ to finish to give $x = 2.303$ & $x = -1.303$

The Ponderal Index is a number which relates a person's height to their weight.

The formula for the Ponderal Index, P , is

$$P = \frac{M}{h^3}$$

where M is weight in kg and h is height in metres.

Find the Ponderal Index for a person who is 1.60 m tall and weighs 72.5 kg.

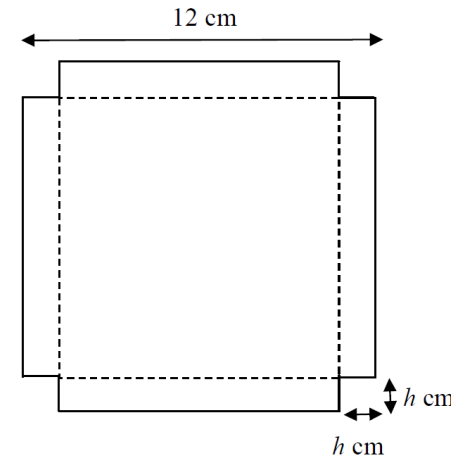
Give your answer correct to 1 decimal place.

$$\begin{aligned} P &= \frac{M}{h^3} \\ P &= \frac{72.5}{(1.6)^3} \\ P &= 17.7 \end{aligned}$$

Algebra to Solve Area and Volume

The diagram below shows a square sheet of cardboard of side length 12 cm, from which four small squares, each of side length h , have been removed. The sheet can be folded to form an open rectangular box of height h .

Show that the volume of the box, in terms of h , is $4h^3 - 48h^2 + 144h$.



$$\begin{aligned} \text{Length} &= 12 - 2h \\ \text{Width} &= 12 - 2h \\ \text{Height} &= h \end{aligned}$$

$$\text{Volume} = l \times w \times h$$

$$\begin{aligned} &= h(12 - 2h)(12 - 2h) \\ &= h(12(12 - 2h) - 2h(12 - 2h)) \\ &= h(144 - 24h - 24h + 4h^2) \\ &= h(144 - 48h + 4h^2) \\ &= 4h^3 - 48h^2 + 144h \end{aligned}$$