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Junior & Leaving Cert

AREA AND VOLUME
TRAPEZOIDAL RULE

Leaving Cert Ordinary level

Ratio and Proportion – Table of Contents

LCOL New Course

LCOL Old Course

A field is divided into eight sections as shown below. The width of each section is 3 metres. The height, in metres, of each section is given in the diagram.

Use the Trapezoidal rule to estimate the area of the field.

Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

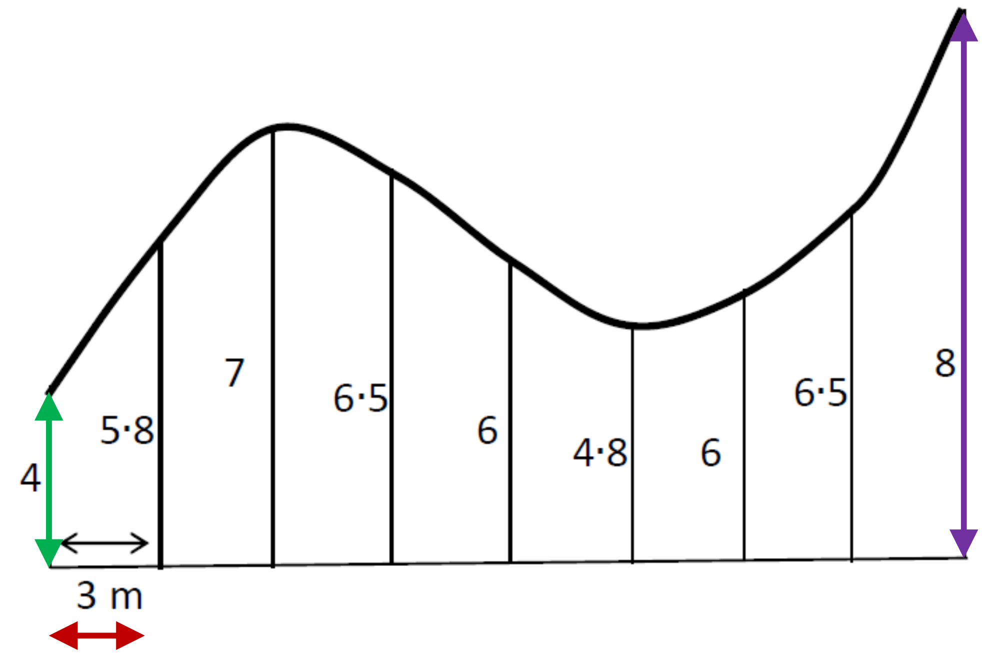
$$A = \frac{3}{2}[4 + 8 + 2(5.8 + 7 + 6.5 + 6 + 4.8 + 6 + 6.5)]$$

$$A = 1.5[12 + 2(42.6)]$$

$$A = 1.5[12 + 85.2]$$

$$A = 1.5[97.2]$$

$$A = 145.8\ \text{m}^2$$

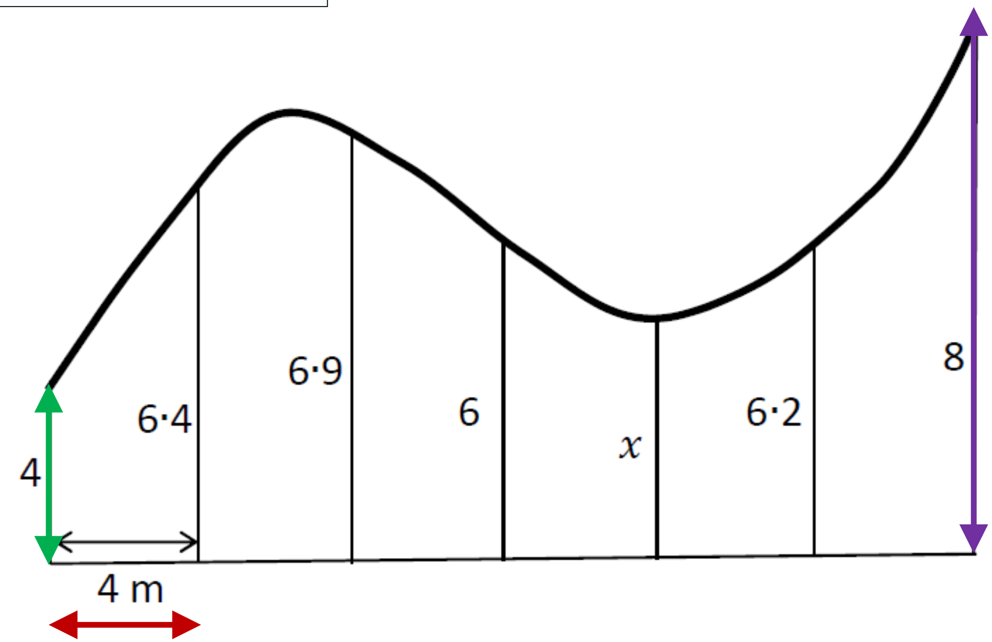


The area of the same field was re-estimated by applying the Trapezoidal rule again. This time, a different section width (4 m) and a different set of section heights were used, as shown below. The area was found to be 145·6 m^2.

Use this information to find the value of the height marked x on the diagram.

Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$



$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$145.6 = \frac{4}{2}[4 + 8 + 2(6.4 + 6.9 + 6 + x + 6.2)]$$

$$145.6 = 2[12 + 2(25.5 + x)]$$

$$145.6 = 2[12 + 51 + 2x]$$

$$145.6 = 2[63 + 2x]$$

$$145.6 = 126 + 4x$$

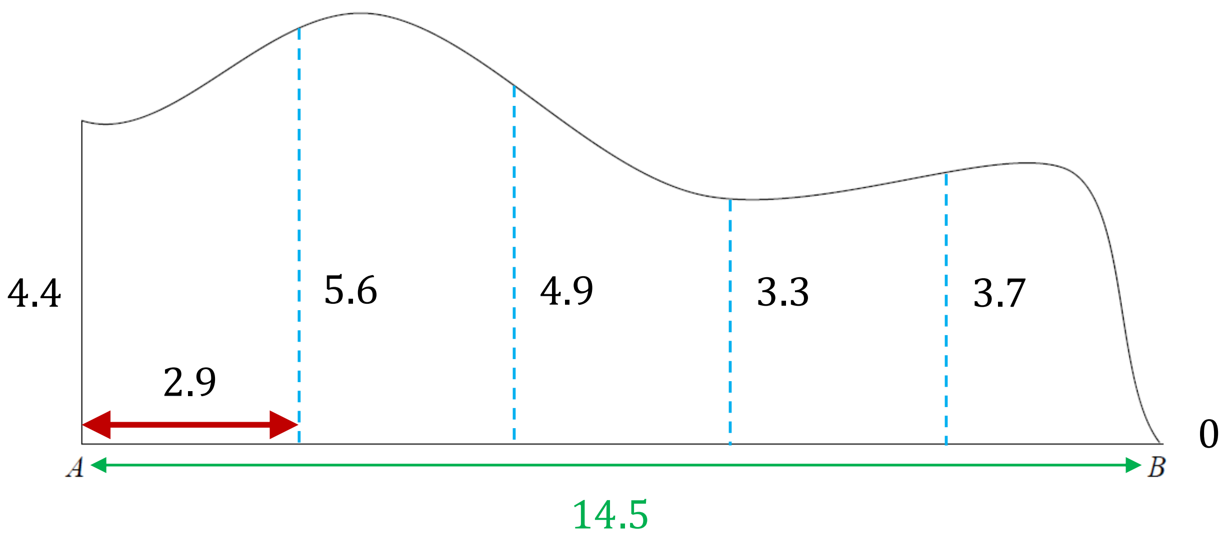
$$19.6 = 4x$$

$$x = 4.9 \text{ m}$$

The diagram below shows a shape with two straight edges and one irregular edge. By dividing the edge $[AB]$ into five equal intervals, use the trapezoidal rule to estimate the area of the shape.

Record your constructions and measurements on the diagram. Give your answer correct to the nearest cm^2.

Must use ruler to measure width and heights. Enter them on the sketch.



$$h = \frac{14.5}{5}$$
$$h = 2.9$$

Trapezoidal Rule

$$A = \frac{h}{2}(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1}))$$

$$\text{A} = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$
$$A = \frac{2.9}{2}[4.4 + 0 + 2(5.6 + 4.9 + 3.3 + 3.7)]$$
$$A = 1.45[4.4 + 2(17.5)]$$
$$A = 1.45[4.4 + 35]$$
$$A = 1.45[39.4]$$
$$A = 57.13\ \text{cm}^2$$

The diagram shows the graph of the function $f(x) = 6x - x^2$ in the domain $0 \leq x \leq 6,\ x \in R$.

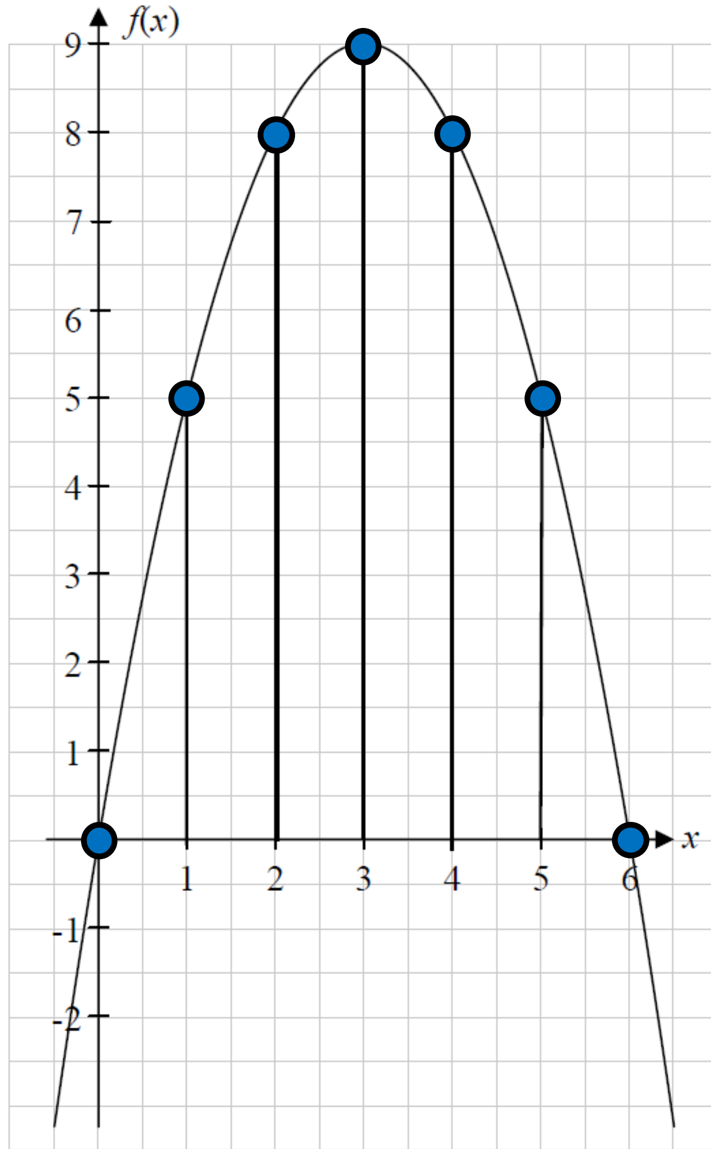
Find $f(0), f(1), f(2), f(3), f(4), f(5)$ and $f(6)$. Hence, complete the table below.

Always fill out a table.

x	0	1	2	3	4	5	6
$f(x)$	0	5	8	9	8	5	0

x	$6x - x^2$	$f(x)$
0	$6(0) - (0)^2$	0
1	$6(1) - (1)^2$	5
2	$6(2) - (2)^2$	8
3	$6(3) - (3)^2$	9
4	$6(4) - (4)^2$	8
5	$6(5) - (5)^2$	5
6	$6(6) - (6)^2$	0

... always check solution by using table mode of the calculator!



Use the trapezoidal rule to estimate the area of the region enclosed between the curve and the x-axis in the given domain.

Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

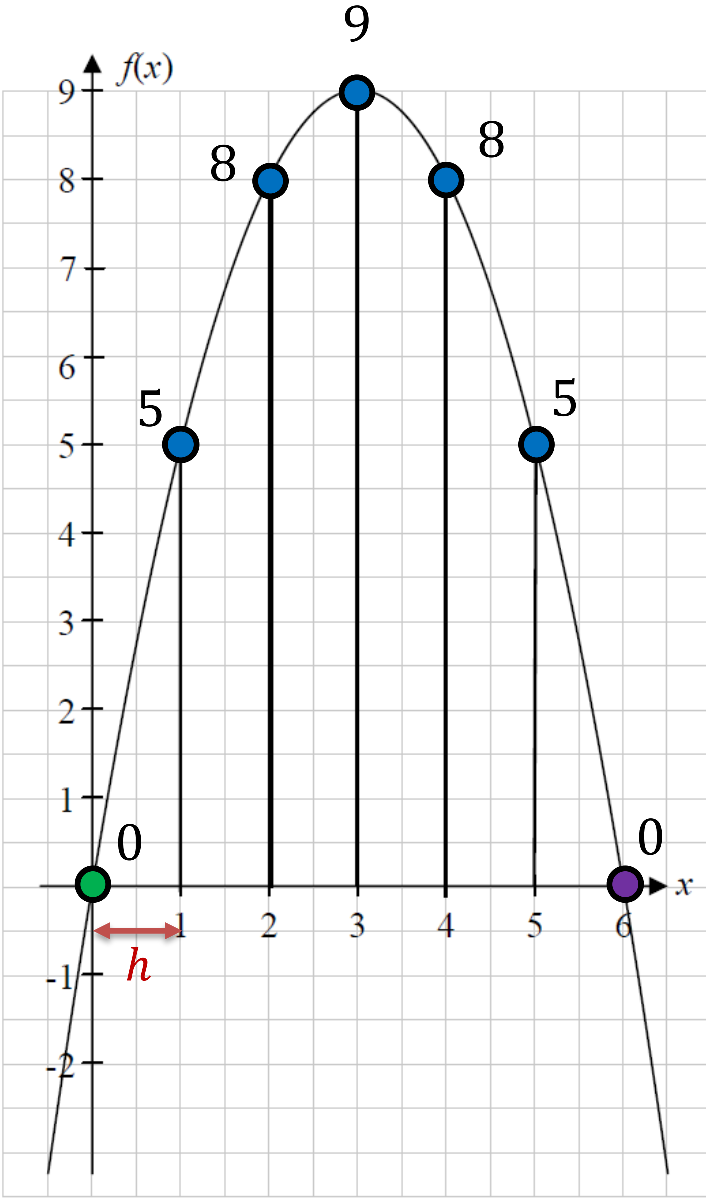
$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$A = \frac{1}{2}\left(0 + 0 + 2(5 + 8 + 9 + 8 + 5)\right)$$

$$A = \frac{1}{2}\left(0 + 2(35)\right)$$

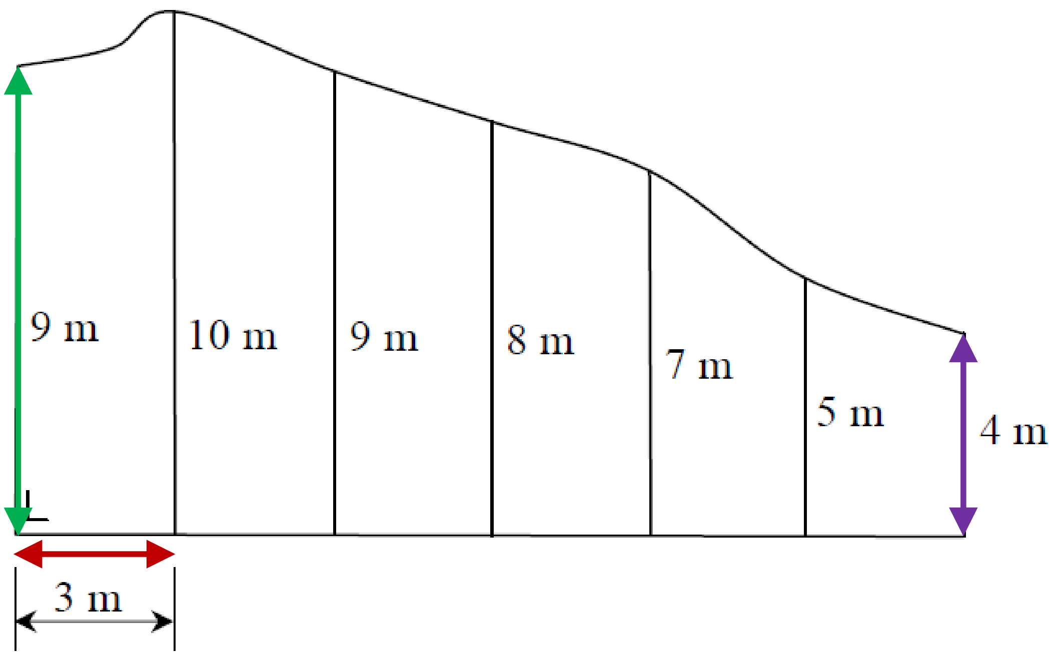
$$A = \frac{1}{2}(70)$$

$$A = 35 \text{ units}^2$$



The sketch shows the garden of a house. At equal intervals of 3 m along one side, perpendicular measurements are made to the boundary, as shown on the sketch.

Use Simpson's Rule **(now the Trapezoidal Rule)** to estimate the area of the garden.



Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$A = \frac{3}{2}\left(9 + 4 + 2(10 + 9 + 8 + 7 + 5)\right)$$

$$A = 1.5\left(13 + 2(39)\right)$$

$$A = 1.5(13 + 78)$$

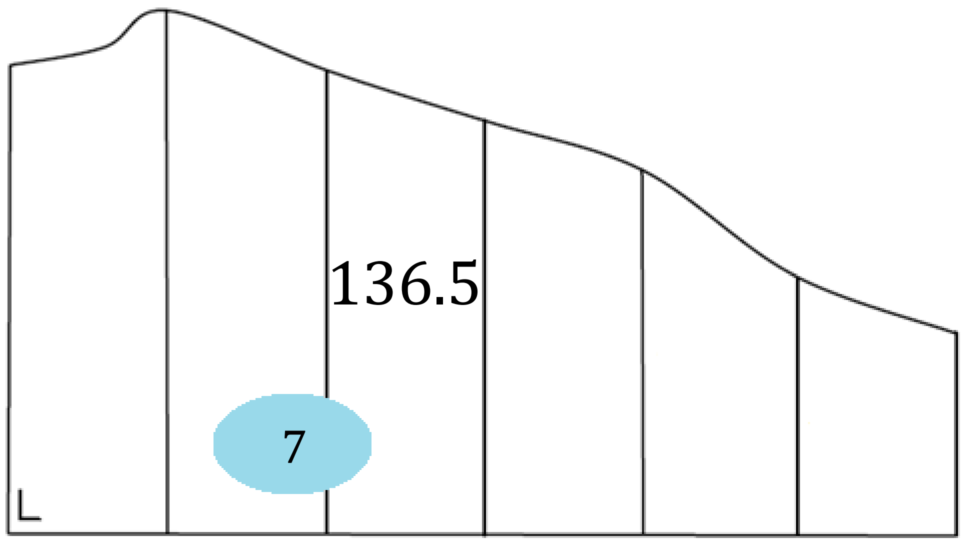
$$A = 1.5(91)$$

$$A = 136.5 \text{ m}^2$$

Question changed from Simpson's Rule as this is no longer on the course.

The owner of the house digs an ornamental pond in the garden. The surface area of the pond is 7 m^2. What percentage of the area of the garden is taken up by the pond?

Give your answer correct to the nearest percent.



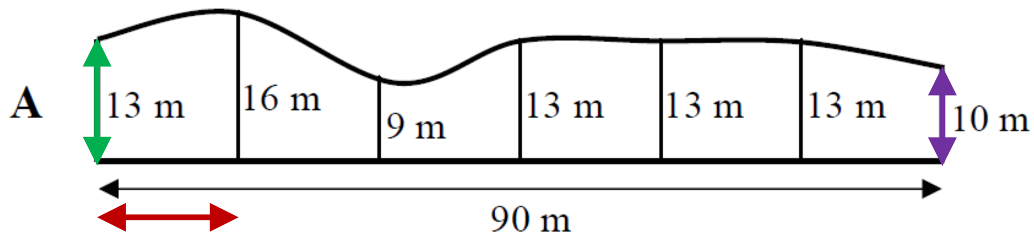
Put the area of the pond over the area of the garden and multiply by 100.

$$\frac{7}{136.5} \times 100$$
$$= 5.13\%$$
$$\approx 5\%$$

Use Simpson's Rule **(now the Trapezoidal Rule)** to determine which of the shapes A or B below has the greater area, and by how much.

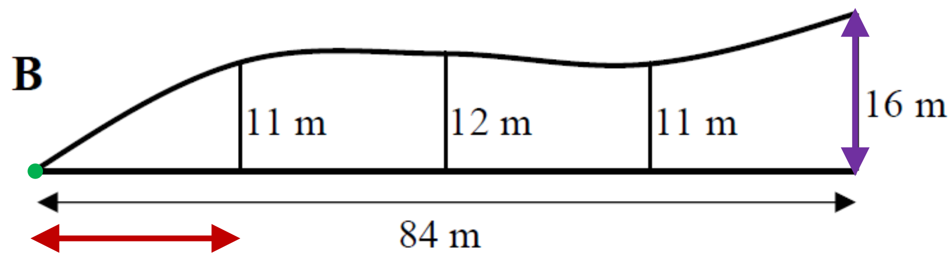
Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$



$$h = \frac{90}{6}$$
$$h = 15$$

$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$
$$A = \frac{15}{2}\left(13 + 10 + 2(16 + 9 + 13 + 13 + 13)\right)$$
$$A = 1132.5 \text{ m}^2$$



$$h = \frac{84}{4}$$
$$h = 21$$

$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$
$$A = \frac{21}{2}\left(0 + 16 + 2(11 + 12 + 11)\right)$$
$$A = 882 \text{ m}^2$$

Thus shape A has the greater area.
It is greater by $1132.5 - 882 = 250.5 \text{ m}^2$

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AREA AND VOLUME
TRAPEZOIDAL RULE

Leaving Cert Ordinary level (Old Course)

The sketch shows a section of a wall that is to be painted.
At equal intervals of 1.2 m along the bottom of the wall, perpendicular measurements are made to the uneven edge, as shown on the sketch.

Use Simpson's Rule **(now the Trapezoidal Rule)** rule to estimate the area of the section of the wall.

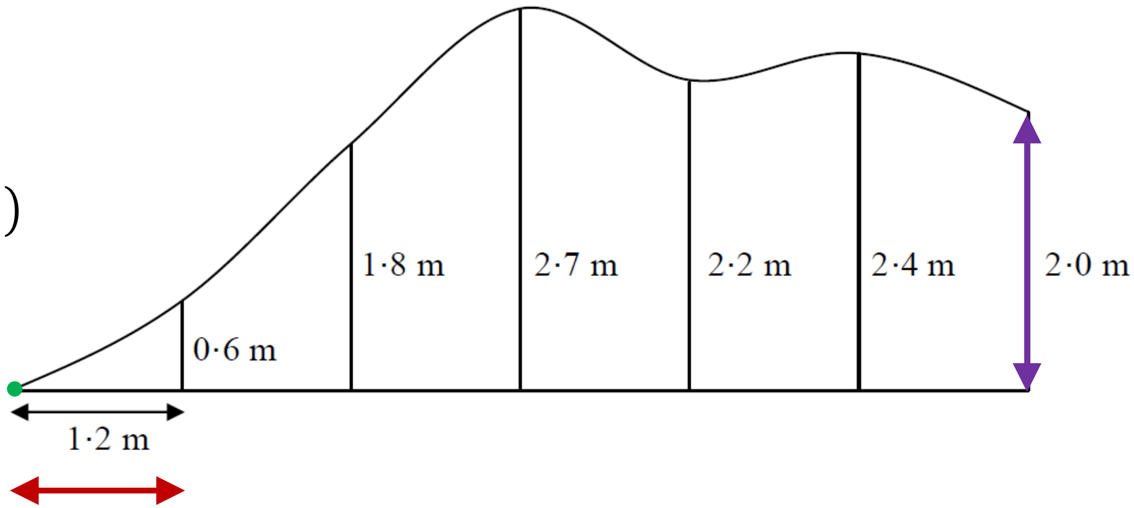
Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$A = \frac{1.2}{2}\left(0 + 2 + 2(0.6 + 1.8 + 2.7 + 2.2 + 2.4)\right)$$

$$A = 12.84 \text{ m}^2$$



(ii)

How many litres of paint are required to paint the section of the wall, if 1 litre of paint covers an area of 2.2 m^2?

Give your answer correct to the nearest litre.

Area = 12.84 m^2
1 litre = 2.2 m^2

$$\frac{12.84}{2.2} = 5.84 \text{ litres}$$

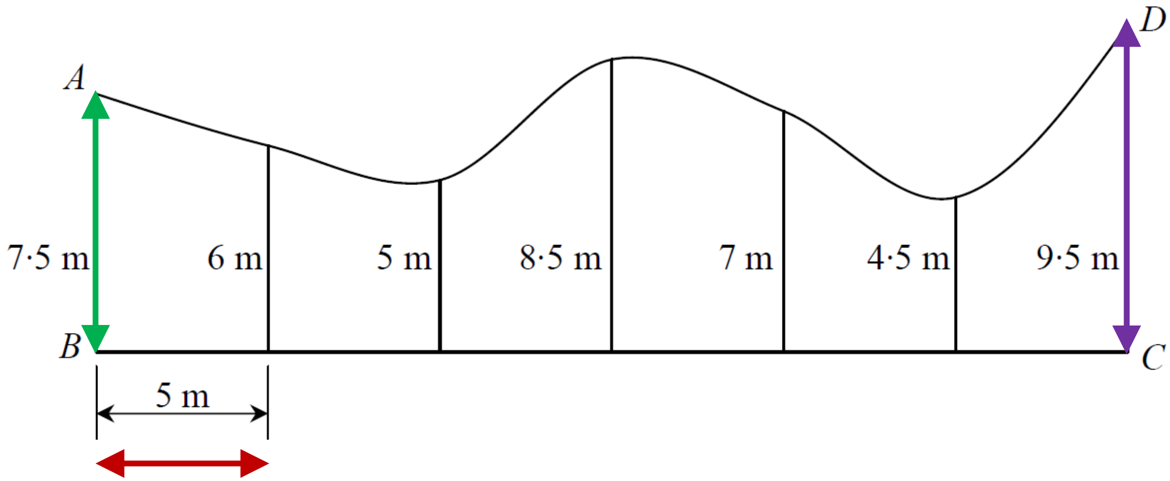
$$\approx 6 \text{ litres}$$

The diagram shows a sketch of a field $ABCD$ that has one uneven edge. At equal intervals of 5 m along , perpendicular measurements are made to the uneven edge, as shown on the sketch.

Use Trapezoidal Rule to estimate the area of the field.

Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$



$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$A = \frac{5}{2}\left(7.5 + 9.5 + 2(6 + 5 + 8.5 + 7 + 4.5)\right)$$

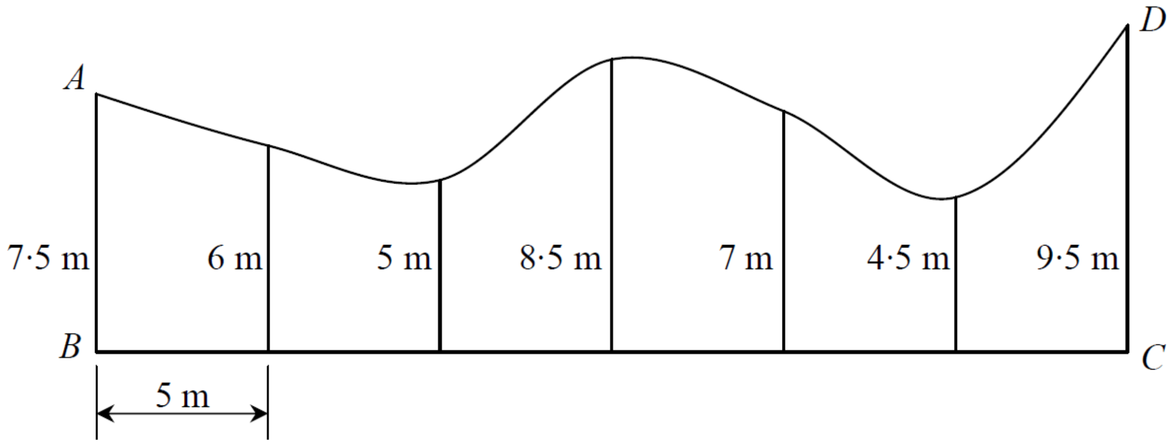
$$A = 197.5 \text{ m}^2$$

The actual area of the field is 200 m^2

Find the percentage error in the estimate.

Percentage Error

$$\frac{\text{Error}}{\text{Actual}} \times 100$$



$$\frac{\text{Error}}{\text{Actual}} \times 100$$

$$= \frac{200 - 197.5}{200} \times 100$$

$$= \frac{2.5}{200} \times 100$$

$$= 1.25\%$$

The sketch shows the garden of a house. At equal intervals of 3 m along one side, perpendicular measurements are made to the boundary, as shown on the sketch.

Use Trapezoidal rule to estimate the area of the garden.

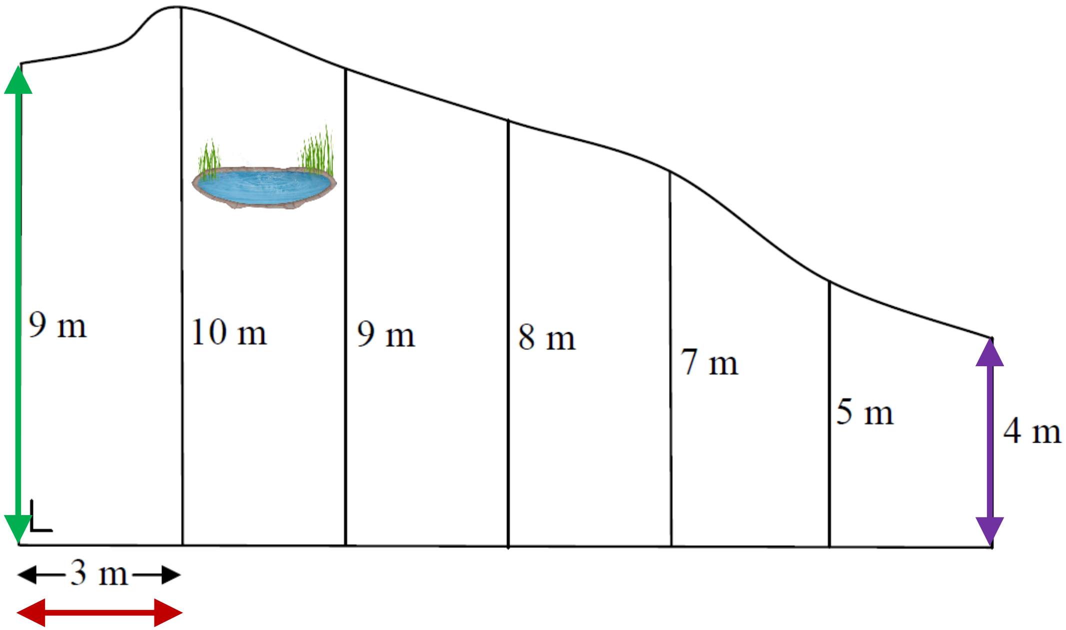
Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$A = \frac{3}{2}(9 + 4 + 2(10 + 9 + 8 + 7 + 5)$$

$$= 136.5 \text{ m}^2$$



(ii)

The owner of the house digs an ornamental pond in the garden. The surface area of the pond is 7 m^2 . What percentage of the area of the garden is taken up by the pond?

Give your answer correct to the nearest percent.

% Taken by Pond

$$= \frac{\text{Area of Pond}}{\text{Area of Garden}} \times 100$$

$$\frac{7}{136.5} \times 100$$

$$= 5.12\%$$

$$\approx 5\%$$

The sketch shows a piece of land which borders the side of a straight road $[ab]$.
The length of $[ab]$ is 54 m.
At equal intervals along $[ab]$, perpendicular measurements are made to the boundary, as shown on the sketch.

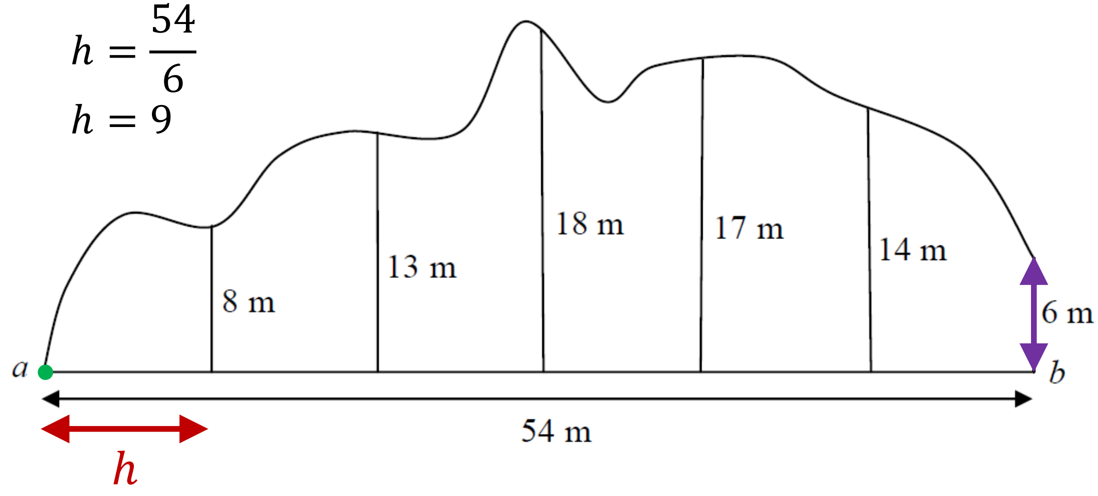
Use Trapezoidal Rule to estimate the area of the piece of land.

Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

$$h = \frac{54}{6}$$

$$h = 9$$



$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$A = \frac{9}{2}(0 + 6 + 2(8 + 13 + 18 + 17 + 14)$$

$$A = 4.5(6 + 2(70)$$

$$A = 4.5(146)$$

$$A = 657\ \text{m}^2$$

(ii)

The land is valued at €480 000 per hectare. Find the value of the piece of land.

Note: 1 hectare = 10 000 m^2 .

$10\,000\ \text{m}^2 = €480{,}000$

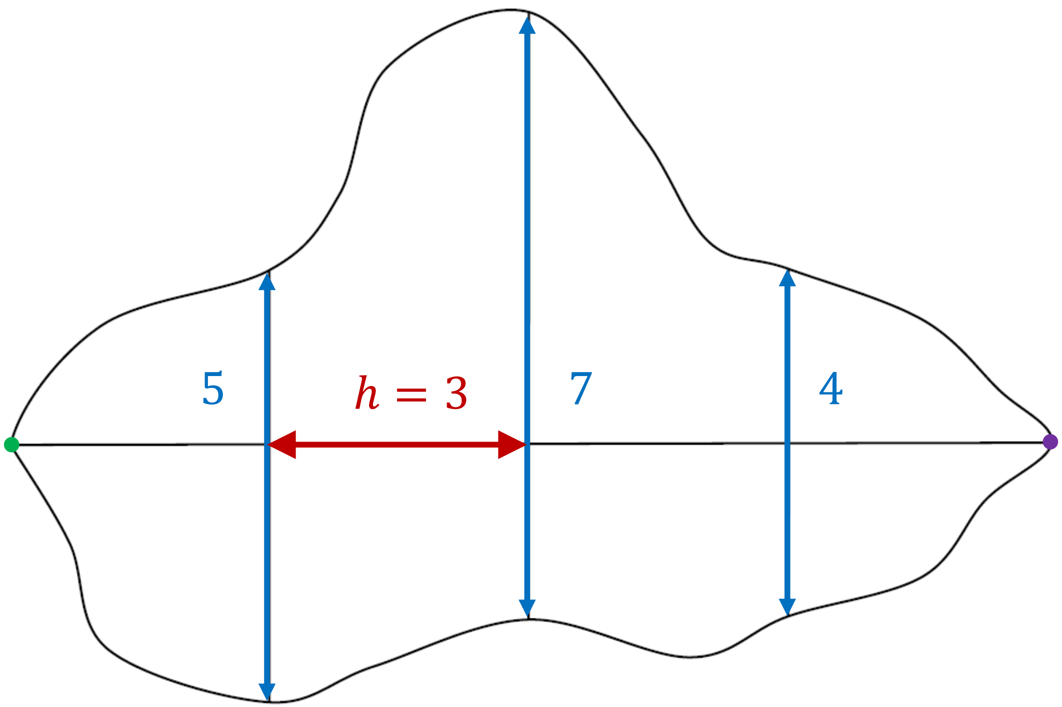
Divide by 10000 and multiply by 657.

$10\,000\ \text{m}^2 = €480{,}000$
$1\ \text{m}^2 = 48$
$657\ \text{m}^2 = 657 \times 48$
$= €31{,}536$

In order to estimate the area of the irregular shape below, a horizontal line is drawn across the widest part of the shape and three offsets (perpendicular lines) are drawn at equal intervals along this line. Measure the horizontal line and the offsets, in centimetres.

Make a rough sketch of the shape in your answerbook and record the measurements on it.

Sometimes you will have to fill in the measurements yourself.



(ii)

Use Trapezoidal Rule with these measurements to estimate the area of the shape.

Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

$$\text{A} = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$

$$\text{A} = \frac{3}{2}\left(0 + 0 + 2(5 + 7 + 4)\right)$$

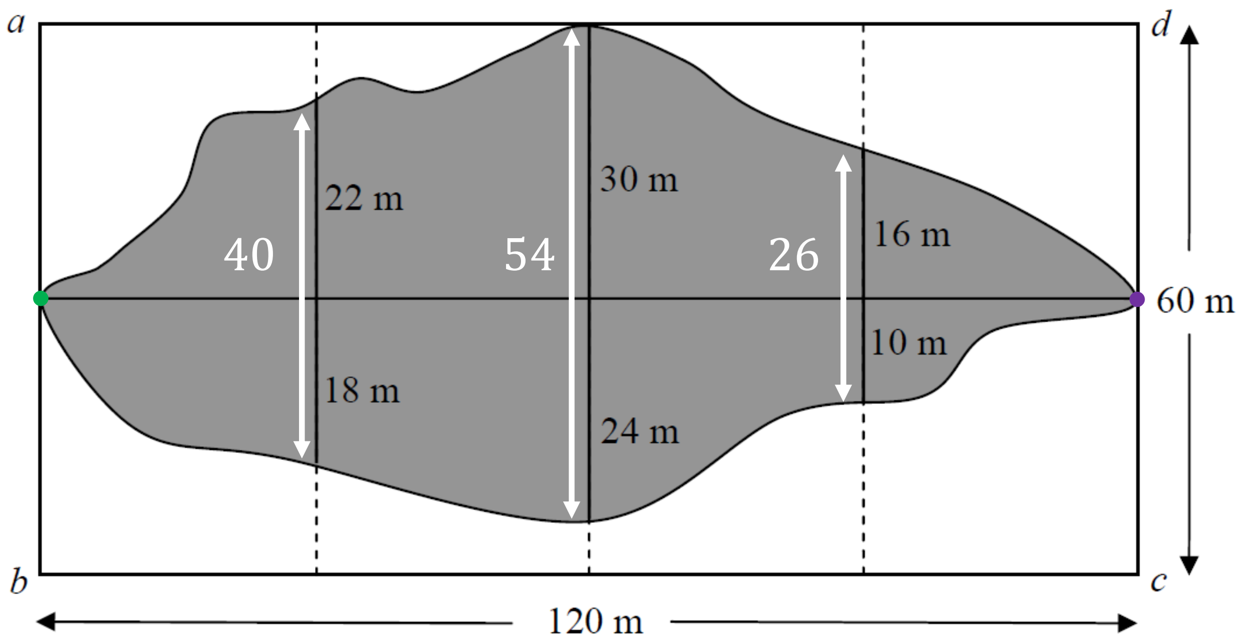
$$\text{A} = 1.5(2(16)$$

$$\text{A} = 1.5(32)$$

$$\text{A} = 48 \text{ cm}^2$$

Archaeologists excavating a rectangular plot *abcd* measuring 120 m by 60 m divided the plot into eight square sections as shown on the diagram. At the end of the first phase of the work the shaded area had been excavated. To estimate the area excavated, perpendicular measurements were made to the edge of the excavated area, as shown.

Use the Trapezoidal Rule to estimate the area excavated.



In this example the heights are the totals from the top of the shape to the bottom.
We also need to calculate h by dividing the full width of the shape by the number of intervals.

$$h = \frac{120}{4}$$
$$h = 30$$

Trapezoidal Rule

$$A = \frac{h}{2}\left(y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\right)$$

$$A = \frac{h}{2}\left(\text{First} + \text{Last} + 2(\text{Sum of the Rest})\right)$$
$$A = \frac{30}{2}\left(0 + 0 + 2(40 + 54 + 26)\right)$$
$$A = 15\left(2(120)\right)$$
$$A = 3600 \text{ m}^2$$

(ii)

Express the excavated area as a percentage of the total area, correct to the nearest whole number.

% Taken by Excavated Area

$$= \frac{\text{Area of Excavated}}{\text{Area of Total}} \times 100$$

$$\frac{3600}{120 \times 60} \times 100$$
$$= 50\%$$