

MATHSPOINTS.IE  
JUNIOR & LEAVING CERT

**ALGEBRA**  
**REFERENCE SHEET**

JUNIOR CERT HIGHER LEVEL

**Simple Equation**

$$3(2x - 1) = 4x$$

$$6x - 3 = 4x$$

$$6x - 4x = 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

Multiply to get rid of brackets, x's to one side, numbers to the other....

**Inequalities**

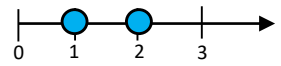
$$5x + 1 \leq 4x + 3, x \in N$$

$$5x - 4x \leq 3 - 1$$

$$x \leq 2$$

Treat this like an equation with x's to one side and numbers to the other.

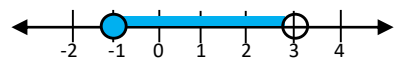
$x \in N, x \in Z$  means dots.  
 $x \in R$  means shading



**Compound Inequalities**

$$-2 \leq 5x + 3 < 18, x \in R$$

Split into two inequalities and solve as before.

$$\begin{array}{ll} -2 \leq 5x + 3 & 5x + 3 < 18 \\ -2 - 3 \leq 5x & 5x < 18 - 3 \\ -5 \leq 5x & 5x < 15 \\ -1 \leq x & x < 3 \end{array}$$


**Substitution**

Calculate the value of  $p^2 - 3rq$   
 For  $p = 3, q = -4$  and  $r = 7$

Sub in values for  $p, q$  and  $r$

$$(3)^2 - 3(7)(-4)$$

$$= 93$$

**Solving Quadratics**

$$f(x) = 2x^2 - 4x - 6 = 0$$

factorise  $(2x - 6)(x + 1) = 0$

or - b formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{16 + 48}}{4}$$

$$x = \frac{4 \pm \sqrt{64}}{4}$$

$$x = \frac{4 \pm 8}{4}$$

$$x = \frac{4 + 8}{4} \quad \& \quad \frac{4 - 8}{4}$$

$$x = 3 \quad \& \quad -1$$

**Forming a Quadratic Equation**

$$x^2 - (\text{sum of the roots})x + (\text{product of the roots}) = 0$$

Form the equation with roots  $x = 4$  and  $x = 7$

$$x^2 - (4 + 7)x + (4)(7) = 0$$

$$x^2 - 11x + 28 = 0$$

Note - 'roots' are values for  $x$  that satisfy the equation.

**Multiplying Brackets (the opposite of Factorising)**

$$(3x + 2)(4x - 3)$$

$$= 3x(4x - 3) + 2(4x - 3)$$

$$= 12x^2 - 9x + 8x - 6$$

$$= 12x^2 - x - 6$$

**Manipulate Formulae (Rearranging)**

This is taking a formula and changing its subject (the letter by itself) through rules of maths. (Multiplication, factorising etc)

$$M = \frac{1}{S + P}$$

Rearrange the formula to make P its subject.

$$M(S + P) = 1$$

$$MS + MP = 1$$

$$MP = 1 - MS$$

$$P = \frac{1 - MS}{M}$$

**Quadratic Graph by Completing the Square**

Express  $f(x) = x^2 + 10x + 32$  in the form  $(x + a)^2 + b$

$$f(x) = x^2 + 10x + 32$$

$$f(x) = x^2 + 10x + \left(\frac{10}{2}\right)^2 + 32 - \left(\frac{10}{2}\right)^2$$

$$f(x) = x^2 + 10x + 25 + 32 - 25$$

$$f(x) = x^2 + 10x + 25 + 7$$

$$f(x) = (x + 5)^2 + 7$$

Min point of curve  $(-5, 7)$  and axis of symmetry is  $x = -5$

**Factorising – 4 types**

**Highest Common Factor**

$$6x^2 - 15xy$$

$$3x(2x - 5y)$$

**Difference of Squares**

$$4x^2 - 81$$

$$(2x + 9)(2x - 9)$$

**Quadratics**

$$6x^2 - 5x - 21$$

$$(3x - 7)(2x + 3)$$

**Grouping**

$$9a^2 - 6ab + 12ac - 8bc$$

$$3a(3a - 2b) + 4c(3a - 2b)$$

$$(3a + 4c)(3a - 2b)$$

**Combinations Simplify**

$$\frac{8x^2 - 12x}{4x^2 - 12x + 9}$$

$$\frac{8x^2 - 12x}{4x^2 - 12x + 9}$$

$$= \frac{4x(2x - 3)}{(2x - 3)(2x + 3)}$$

$$= \frac{4x}{2x + 3}$$

### Laws of Indices

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^n} = \frac{1}{n\sqrt[n]{a}} \quad \left| \quad \frac{m}{a^n} = \sqrt[n]{a^m}$$

$$a^0 = 1$$

### Indices Example

$$\frac{8^x}{\sqrt{2}} = 4^{2x+1}$$

Express all the terms as powers of 2.

$$\frac{(2^3)^x}{2^{\frac{1}{2}}} = (2^2)^{2x+1}$$

$$\frac{2^{3x}}{2^{\frac{1}{2}}} = 2^{4x+2}$$

$$2^{3x-\frac{1}{2}} = 2^{4x+2}$$

$$3x - \frac{1}{2} = 4x + 2$$

$$2 - \frac{1}{2} = 4x - 3x$$

$$1\frac{1}{2} = x$$

### Writing Expressions/Solving Problems

The length of a rectangle is 5 times its width. The perimeter of the rectangle is 120m.

width =  $x$       length =  $5x$   
 $x + x + 5x + 5x = 120$   
 $x = 10$

### Properties of Surds

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a}\sqrt{a} = a$$

### Surds Example

$$(x + \sqrt{x})(x - \sqrt{x}) = 6$$

$$x(x - \sqrt{x}) + \sqrt{x}(x - \sqrt{x}) = 6$$

$$x^2 - x\sqrt{x} + x\sqrt{x} - \sqrt{x}^2 = 6$$

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3 \quad x = -2$$

**not a solution**

### Verify

This means check your solutions.

From above:  
 $x = 3$   
 $(3 + \sqrt{3})(3 - \sqrt{3}) = 6$   
 $9 + 3\sqrt{3} - 3\sqrt{3} - \sqrt{3}^2 = 6$   
 $9 - 3 = 6$

**Verified**

### Express as a Single Fraction

Write as a single fraction

$$\frac{3x-2}{4} + \frac{2x}{3} - \frac{5x-1}{12}$$

Find a common denominator.

$$= \frac{3(3x-2) + 4(2x) - 1(5x-1)}{12}$$

$$= \frac{12x-5}{12}$$

### Equations Using Fractions

$$\frac{2}{3x-4} - \frac{1}{2x+1} = \frac{1}{2}$$

Similar to above but we can drop the denominator. Remember to put any whole number over 1.

$$\frac{2(2)(2x+1) - 1(2)(3x-4)}{2(3x-4)(2x+1)} = \frac{1}{2}$$

$$8x + 4 - 6x + 8 = 6x^2 - 8x + 3x - 4$$

$$0 = 6x^2 - 8x - 8x + 6x + 3x - 8 - 4 - 4$$

$$6x^2 - 7x - 16 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-16)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{433}}{12}$$

$$x = \frac{7 + \sqrt{433}}{12} \quad \& \quad x = \frac{7 - \sqrt{433}}{12}$$

$$x = 2.3 \quad \& \quad -1.2$$

**Algebra – Junior Cert Higher Level Reference Sheet**

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### Simultaneous Equations – 2 unknowns (linear)

Solve

$$4x + 16y = 20$$

$$2x - 3y = -1$$

Multiply one or both lines to make co-efficients of one of the variables the same. Cancel down and solve.

$$4x + 16y = 20$$

$$2x - 3y = -1 \quad \times -2 \quad \frac{4x + 6y = 2}{-4x + 6y = 2}$$

$$22y = 22$$

$$y = 1$$

←

$$4x + 16y = 20$$

$$4x + 16(1) = 20$$

$$4x + 16 = 20$$

$$4x = 4$$

$$x = 1$$

Also occurs in co-ordinate geometry to find where lines intersect.

### Algebra Division

Divide  $2x^3 + x^2 - 13x + 6$  by  $x + 3$

$$\begin{array}{r}
 2x^2 - 5x + 2 \\
 x+3 \overline{) 2x^3 + x^2 - 13x + 6} \\
 \underline{-2x^3 + 6x^2} \phantom{+ 6} \\
 8x^2 - 13x + 6 \\
 \underline{-8x^2 + 24x} \phantom{+ 6} \\
 37x + 6 \\
 \underline{-37x - 111} \\
 -105
 \end{array}$$

### Writing Expressions/Solving Problems II

$x$  is a real number.  
 One new number is formed by increasing  $x$  by 1.  
 A second new number is formed by decreasing  $x$  by 2.  
 The product of the 2 new numbers is 1  
 Find  $x$ .

$$(x+1)(x-2) = 1$$

$$x(x-2) + 1(x-2) = 1$$

$$x^2 - 2x + x - 2 - 1 = 0$$

$$x^2 - x - 3 = 0$$

Use  $-b$  to finish to give  $x = 2.303$  &  $x = -1.303$