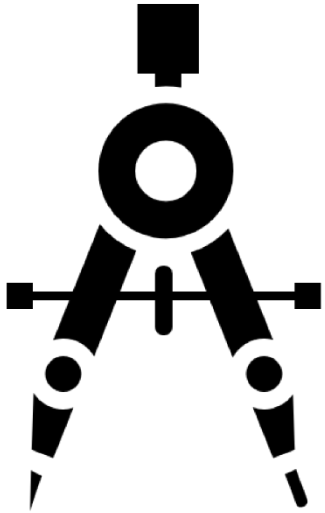


MATHSPOINTS.IE

JUNIOR & LEAVING CERT

1.4 LINEAR ACCELERATION

$$F = ma$$

LEAVING CERT APPLIED MATHS HIGHER LEVEL

1.4 Linear Acceleration: Using $F = ma$

Table of Contents

A bullet of mass m is fired with speed v into a fixed block of wood and is brought to rest in a distance d. Find the resistance to motion assuming it to be constant.
Another bullet also of mass m is then fired with speed $2v$ into another fixed block of thickness $2d$, which offers the same resistance as the first block. Find the speed with which the bullet emerges, and the time it takes to pass through the block.

$v = u + at$
$v^2 = u^2 + 2as$
$s = ut + \frac{1}{2}at^2.$

Calculate the acceleration of the object in terms of v and d.

$u = v$
$v = 0$
$a = ?$
$t = ?$
$s = d$

$v^2 = u^2 + 2as$
$0^2 = v^2 + 2ad$
$0 = v^2 + 2ad$
$a = -\frac{v^2}{2d}\ \text{m/s}^2$

Calculate the resistance of the block of wood.

$F = ma$

$F = T - R$
Where T is the tractive force and R is the resistance.

$-R = m\left(-\frac{v^2}{2d}\right)$
$R = \frac{mv^2}{2d}$

Find x, the speed of the bullet exiting the block.

Use x instead of v as v is used already.

$u = 2v$
$v = x$
$a = -\frac{v^2}{2d}$
$t = ?$
$s = 2d$

$v^2 = u^2 + 2as$
$x^2 = (2v)^2 + 2\left(-\frac{v^2}{2d}\right)(2d)$
$x^2 = 4v^2 - 2v^2$
$x^2 = 2v^2$
$x = \sqrt{2}v$

$u = 2v$
$v = \sqrt{2}v$
$a = -\frac{v^2}{2d}$
$t = ?$
$s = 5s$

$v = u + at$
$\sqrt{2}v = 2v - \frac{v^2}{2d}t$
$\frac{v^2}{2d}t = v(2 - \sqrt{2})$
$t = \frac{2d(2-\sqrt{2})}{v}$ seconds

A particle of mass 3 grammes falls from rest from a height of 0.4 m on to a soft material into which it sinks 0.0245 m.
Neglecting air resistance, calculate the constant resistance of the material.

$v = u + at$
$v^2 = u^2 + 2as$
$s = ut + \frac{1}{2}at^2.$

Calculate the speed of the object at the moment in hits the soft material.

$u = 0$
$v = ?$
$a = 9.8$
$t = t$
$s = 0.4$

$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2(9.8)(0.4)$
$v^2 = 7.84$
$v = 2.8 \text{ m/s}$

Calculate the acceleration of the object in the soft material given that it will stop when $v = 0$.

$u = 2.8$
$v = 0$
$a = a$
$t = t$
$s = 0.0245$

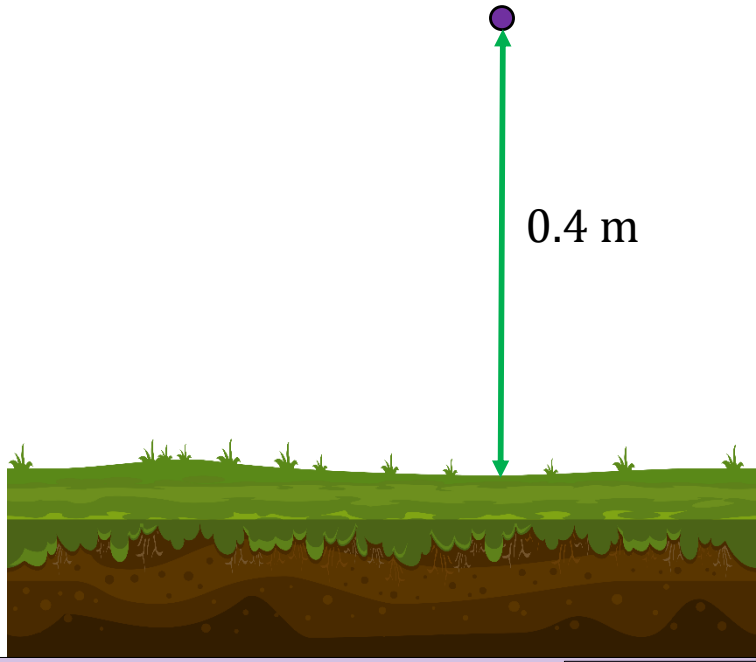
$v^2 = u^2 + 2as$
$0^2 = (2.8)^2 + 2a(0.0245)$
$0 = 7.84 + 0.049a$
$a = -160 \text{ m/s}^2$

Calculate the resistance of the material.

$F = ma$

$W - R = 0.003(-160)$
$0.003(9.8) - R = 0.003(-160)$
$0.0294 - R = -0.48$
$R = 0.5094 \text{ N}$

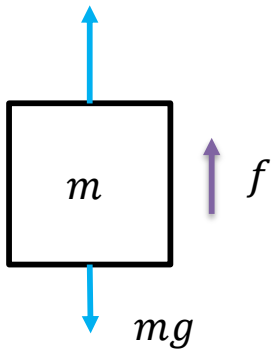
$F = W - R$
Where W is the weight and R is the resistance.



In a lift, moving upwards with acceleration f, a spring balance indicates an object to have a weight of 98 N. When the lift is moving downwards with acceleration $2f$ the weight appears to be 68.6 N.
Calculate
(i) the actual weight .
(ii) the downward acceleration of the lift.

Form equations for both situations.

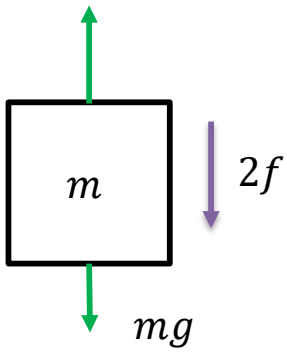
Upwards



$F = ma$

$98 - mg = mf$
$mg + mf = 98$

Downwards



$F = ma$

$mg - 68.6 = 2mf$
$mg - 2mf = 68.6$

Solve simultaneous equation.

$2mg + 2mf = 196$
$mg - 2mf = 68.6$
$3mg = 264.6$
$mg = 88.2$
$m = 9 \text{ kg}$

→ Actual Weight

$mg + mf = 98$
$9g + 9f = 98$
$9f = 98 - 9g$
$f = \dfrac{98 - 9g}{9}$
$f = \dfrac{49}{45}$

Downward Acceleration

$2f = \dfrac{98}{45} \text{ m/s}^2$

A car of mass 1500 kg travels up a slope of gradient $\sin^{-1}\left(\frac{1}{50}\right)$ against a constant resistance of 0.2 N per kilogram.

Find

(i) the constant force required to produce an acceleration of 0.1 m/s^2.

(ii) the power which is developed when the speed is 20 m/s.

(i)

Equation of motion up the incline.

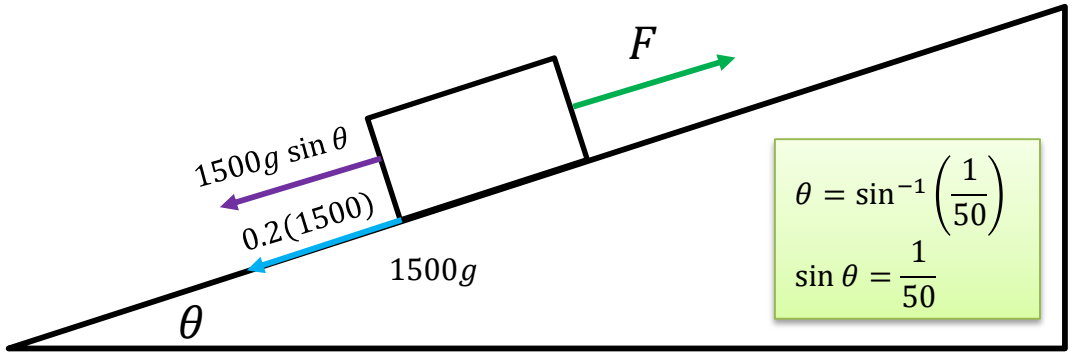
$F = ma$

$T - 0.2(1500) - 1500g(\sin\theta) = 1500(0.1)$

$T - 300 - 1500g\left(\frac{1}{50}\right) = 150$

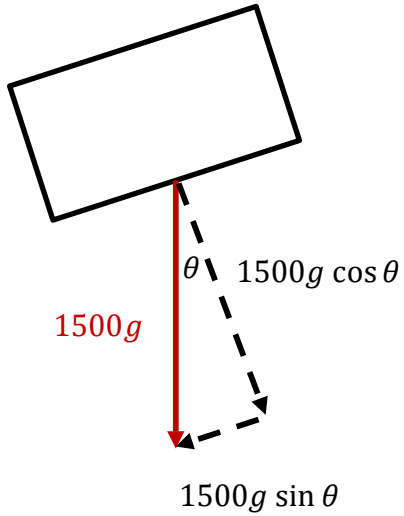
$T - 300 - 294 = 150$

$T = 744$ N



Force Diagram

Resolve the vector into components parallel and perpendicular to the incline.



(ii)

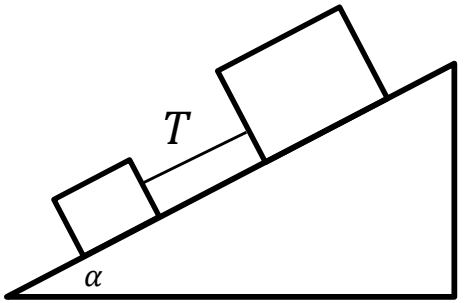
Power $= Tv$

$= (744)(20)$

$= 14880$ W

A car of mass 1200 kg tows a caravan of mass 900 kg first along a horizontal road with acceleration f and then up an incline α to the horizontal road at uniform speed.
The force exerted by the engine is 2700 N. Friction and air resistance amount to 150 N on the car and 240 N on the caravan.

(i) Calculate the acceleration, f, of the car along the horizontal road.
(ii) Calculate the value of α, to the nearest degree.



(i)

First calculate the acceleration along horizontal road.

$F = ma$

$2700 - 150 - 240 = 2100f$
$f = 1.1 \text{ m s}^{-2}$

Form equations of motion up the incline.

$F = ma$

Car.

$2700 - 150 - T - 1200g \sin\alpha = 1200(0)$
$-T - 1200g \sin\alpha = -2550$

Caravan.

$T - 900g \sin\alpha - 240 = 900(0)$
$T - 900g \sin\alpha = 240$

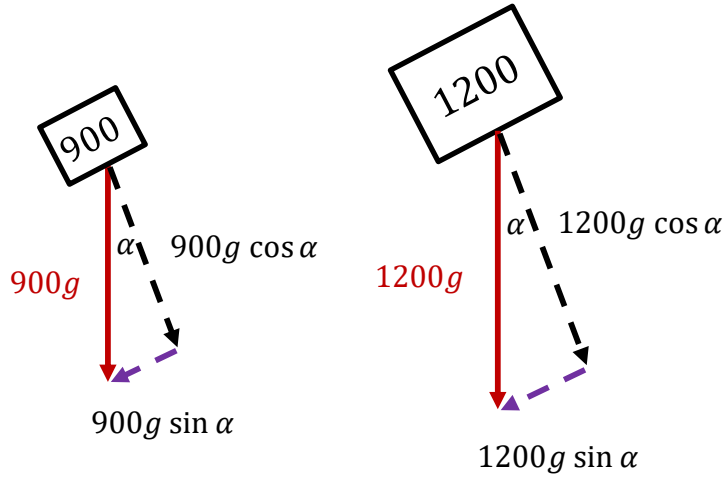
(ii)

Solve the simultaneous equation.

$-T - 1200g \sin\alpha = -2550$
$T - 900g \sin\alpha = 240$
$-2310g \sin\alpha = -2550$
$\sin\alpha = \dfrac{2550}{2310g}$
$\alpha = 6.46766°$
$\alpha \approx 6°$

Uniform Speed means no acceleration!

Force Diagrams



A mass of 8 kg falls freely from rest. After 5 s the mass penetrates sand. The sand offers a constant resistance and brings the mass to rest in 0.01 s.

(i) Find the constant resistance of the sand

(ii) Find the distance the mass penetrates into the sand.

$v = u + at$
$v^2 = u^2 + 2as$
$s = ut + \frac{1}{2}at^2.$

(i)

Calculate the speed of the mass as it hits the sand ($t = 5$).

$u = 0$
$v = ?$
$a = 9.8$
$t = 5$
$s = ?$

$v = u + at$
$v = 0 + 9.8(5)$
$v = 49$ m/s

Use this to find the deceleration in the sand.

$u = 49$
$v = 0$
$a = ?$
$t = 0.01$
$s = ?$

$v = u + at$
$0 = 49 + a(0.01)$
$0.01a = -49$
$a = -4900$

Use equation of motion to calculate the resistance.

$F = ma$

$F = W - R$
Where W is the weight and R is the resistance.

$mg - R = ma$
$8(9.8) - R = 8(-4900)$
$78.4 - R = -39200$
$R = 39278.4$ N

(ii)

Calculate the distance travelled in the sand before stopping.

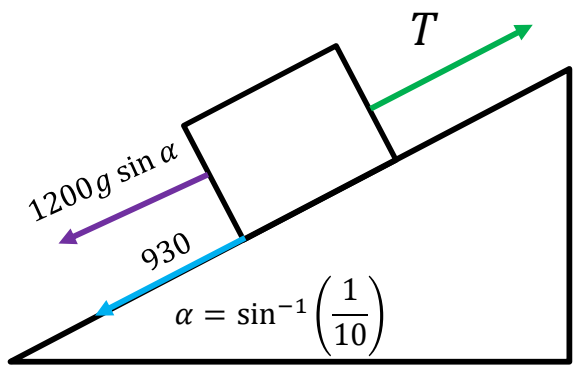
$u = 49$
$v = 0$
$a = -4900$
$t = t$
$s = s$

$v^2 = u^2 + 2as$
$0^2 = 49^2 + 2(-4900)s$
$0 = 2401 - 9800s$
$9800s = 2401$
$s = 0.245$ m

At a particular instant a car of mass 1200 kg is towing a trailer of mass 450 kg on a level road at a speed of 25 m s^{-1} when the engine exerts a constant power of 50 kW. Friction and air resistance amount to 930 N on the car and 200 N on the trailer.

(i) Find the acceleration of the car at this instant.

(ii) Calculate the maximum speed at which the car (without the trailer) could travel up an incline of $\sin^{-1}\frac{1}{10}$ against the same resistance with the engine working at the same rate.



(i)

The formula for Power is tractive force multiplied by velocity.
Us this and the information given to find, T, the tractive force.

Power $= Tv$

Note
50 kW = 50000 W

$$Tv = 50000$$
$$T = \frac{50000}{v}$$
$$T = \frac{50000}{25}$$
$$T = 2000$$

Use Equation of Motion to find acceleration.

$F = ma$

$$2000 - 930 - 200 = 1650a$$
$$a = 0.527 \text{ m s}^{-2}$$

(ii)

Moving uphill, gravity acts on the car.

$F = ma$

Max Speed means no acceleration!

$$T - 930 - 1200g\sin\alpha = 1200(0)$$
$$T - 930 - 1200g\left(\frac{1}{10}\right) = 1200(0)$$
$$T = 2106$$

$P = Tv$

$$50000 = 2106v$$
$$v = \frac{50000}{2106}$$
$$= 23.74 \text{ m s}^{-1}$$

Force Diagram