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Junior & Leaving Cert

1. Linear Motion

Leaving Cert Applied Maths

1. Linear Motion

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If a graph is drawn of **velocity** (y-axis) against **time** (x-axis) then:

Distance Travelled = Area Under Graph

It's often quicker to use this than to use the equation

$$s = ut + \frac{1}{2}at^2$$

Break the area under the graph into triangles and rectangles and sum the areas of each to find the total distance travelled.

The slope of the line for a given stage corresponds to the acceleration of the object during that stage.

This can also be quicker than using formula.

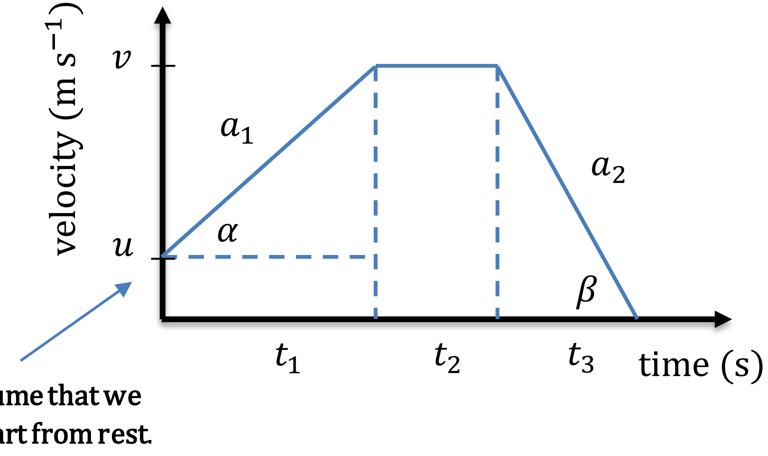
When the particle is accelerating the graph is diagonal upwards.
When the particle is at constant speed the graph goes straight across.
When the particle is decelerating the particle is diagonal downwards.

If asked for the average speed of the journey use:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed} = \frac{\text{sum of areas under graph}}{t_1 + t_2 + t_2}$$

Note that the area between the graph and the time axis represents the displacement s of the body.



Acceleration is the slope of the line.

$$a_1 = \tan\alpha = \frac{\text{rise}}{\text{run}}$$

Deceleration

$$a_2 = \tan\beta = \frac{\text{rise}}{\text{run}}$$

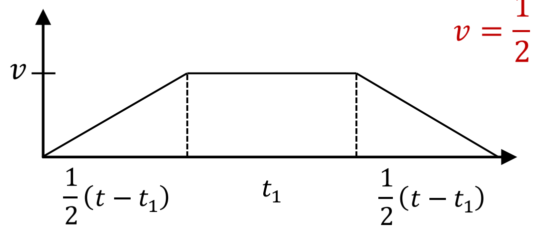
To solve complex problems we need to be comfortable expressing some variables in terms of the others so that we can reduce the amount of variables we have to work with.

(see 2013 (b)). Show that $v = \frac{1}{2}f(t - t_1)$

How would you solve the problem if the letters were numbers and proceed in the same way.

If f is the acceleration then $f = \dfrac{v}{\frac{1}{2}(t - t_1)}$

$$v = \frac{1}{2}f(t - t_1)$$



Revision Checklist

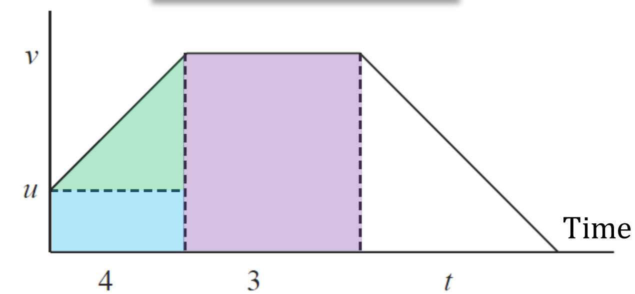
- 2016 (a) ☐
- 2013 (b) ☐
- 2011 (b) ☐
- 2007 (b) ☐
- 1999 (b) ☐
- 1998 (a) ☐
- 1997 (a) ☐
- 1996 (b) ☐
- 1991 (a) ☐
- 1990 (b) ☐
- 1986 (a) ☐
- 1979 (a) ☐
- 1977 ☐
- 1972 ☐

A car has an initial speed of u m s^{-1}. It moves in a straight line with constant acceleration f for 4 seconds. It travels 40 m while accelerating.
The car then moves with uniform speed and travels 45 m in 3 seconds.
It is then brought to rest by a constant retardation $2f$.

(i) Draw a speed-time graph for the motion.
(ii) Find the value of u.
(iii) Find the total distance travelled.

(i)

Speed-Time Graph



Speed $=\dfrac{\text{Distance}}{\text{Time}}$

(ii)

$v=\dfrac{45}{3}$
$v=15$ m/s

or

Area of Rectangle
$3v=45$
$v=15$ m/s

Travels 40 while accelerating.
Area of Rectangle + Area of Triangle = 40

$4u+\frac{1}{2}(4)(v-u)=40$
$4u+2(15-u)=40$
$4u+30-2u=40$
$2u=10$
$u=5$ m/s

(iii)

Calculate f using information during the acceleration section.

$u=5$
$v=15$
$a=f$
$t=4$

$v=u+at$
$15=5+f(4)$
$10=4f$
$f=2.5 \text{ m s}^{-2}$

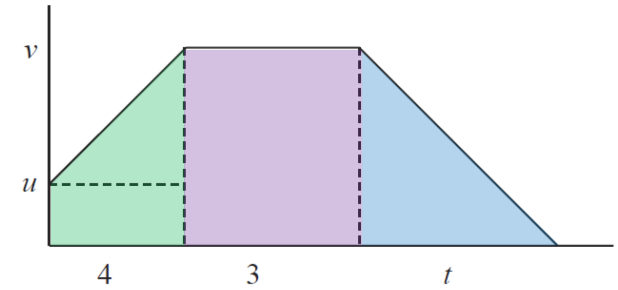
Can be quicker to find accelerations using:

$f=\tan\alpha=\dfrac{10}{4}$
$f=2.5 \text{ m s}^{-2}$

Calculate t using information during the deceleration section.

$u=15$
$v=0$
$a=-2f$
$t=t$

$v=u+at$
$0=15+(-2f)(t)$
$0=15+\big(-2(2.5)\big)(t)$
$0=15-5t$
$t=3$ seconds



Distance = Area Under Curve

$=40+45+\frac{1}{2}(3)(15)$
$=40+45+22.5$
$=107.5$ m

Some velocity time graphs have no stage with constant velocity.
The object accelerates and then immediately decelerates.
If the acceleration is equal to the deceleration then the time taken will be the same.

If they are not equal then the ratio of acceleration to deceleration is opposite to the ratio of the times taken.

$t_1 : t_2 = d : a$

To be sure of full marks generate this formula by getting the acceleration and deceleration in terms of v and letting the v's equal. See top right!

We can use this information to express each of the times t_1 and t_2 as a fraction of the total time T.

We also try and express the velocity, v in terms of the T.

As with all topics in Applied Maths we need to be comfortable working with letters instead of numbers.
The theory is the same but the finish can be trickier to spot (see 2006 Q1 (a)).

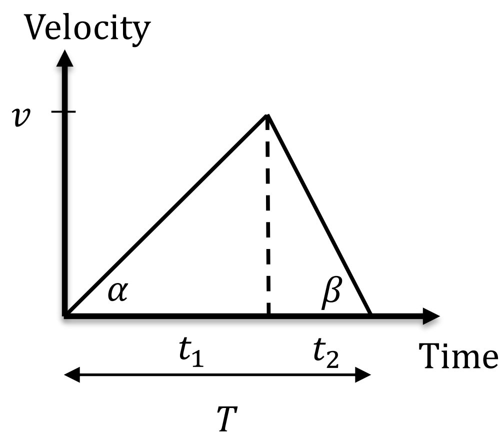
When labelling your graph include velocity v, times for acceleration and deceleration t_1 and t_2 and a total time, T.

Also include the angles α and β.

The maximum acceleration of a body is 4 m/s^2 and its maximum retardation is 8 m/s^2.

What is the shortest time in which the body can travel a distance of 1200 m from rest to rest?

Velocity – Time Graph



$t_1 : t_2 = d : a$

Find expressions for t_1 and t_2 in terms of T.

$$t_1 : t_2 = d : a$$
$$t_1 : t_2 = 8 : 4$$
$$= \frac{2}{3} : \frac{1}{3}$$

$$t_1 = \frac{2}{3}T$$
$$t_2 = \frac{1}{3}T$$

Alternate method to find ratio of times.

$$\tan\alpha = \frac{v}{t_1}$$
$$4 = \frac{v}{t_1}$$
$$v = 4t_1$$

$$\tan\beta = \frac{v}{t_2}$$
$$8 = \frac{v}{t_2}$$
$$v = 8t_2$$

$$t_1 = 2t_2$$

Find expressions for v in terms of T.

$$\tan\alpha = \frac{v}{t_1} = 4$$
$$v = 4t_1$$
$$v = 4\left(\frac{2}{3}T\right)$$
$$v = \frac{8}{3}T$$

Distance = Area Under Graph

$$\frac{1}{2}T(v) = 1200$$
$$\frac{1}{2}(T)\left(\frac{8}{3}T\right) = 1200$$
$$\frac{4}{3}T^2 = 1200$$
$$T^2 = 900$$
$$T = 30 \text{ s}$$

Revision Checklist

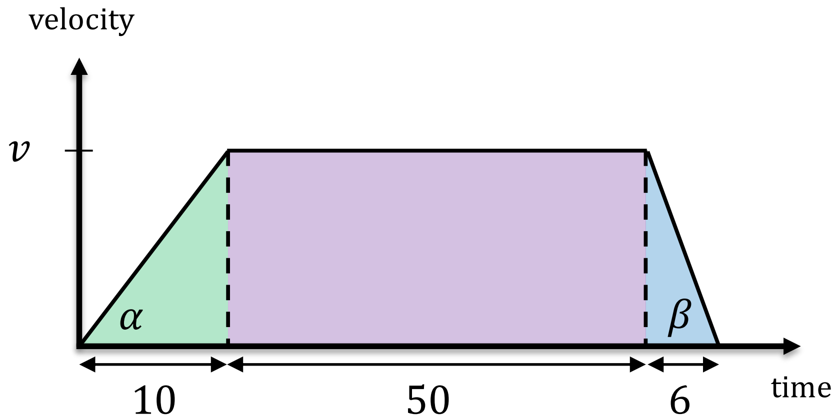
- 2009 (b) ☐
- 2006 (a) ☐
- 2001 (a) ☐
- 1994 (a) ☐
- 1987 (a) ☐
- 1981 ☐
- 1979 (b) ☐
- 1978 ☐
- 1973 ☐

A body starts from rest at p, travels in a straight line and then comes to rest at q which is 0.696 km from p. The time taken is 66 seconds.
For the first 10 seconds if has uniform acceleration $a1$.
It then travels at constant speed and is finally brought to rest by a uniform deceleration $a2$ acting for 6 seconds.

(i) Calculate a_1 and a_2.

(ii) If the journey from rest at p to rest at q had been travelled with no interval of constant speed, but subject to a_1 for time t_1 followed by a_2 for time t_2, show that time for the journey is $8\sqrt{29}$ seconds.

(i)



First calculate v.

Distance = Area Under Graph

$$\frac{1}{2}(10)(v) + (50)(v) + \frac{1}{2}(6)(v) = 696$$
$$5v + 50v + 3v = 696$$
$$58v = 696$$
$$v = 12$$

Calculate the accelerations using tan or formulae.
Note: tan method quicker.

$$a = \tan\alpha = \frac{12}{10}$$
$$a_1 = 1.2 \text{ m/s}^2$$

$$d = \tan\beta = \frac{12}{6}$$
$$a_2 = 2 \text{ m/s}^2$$

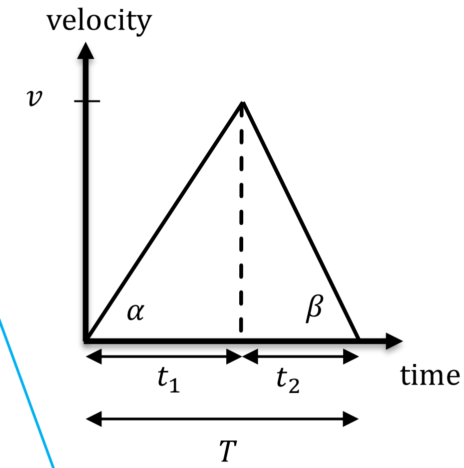
(ii)

Find expressions for t_1 and t_2 in terms of T.

$$t_1 : t_2 = d : a$$
$$t_1 : t_2 = 2 : 1.2$$
$$= 5 : 3$$

$$t_1 = \frac{5}{8}T$$
$$t_2 = \frac{3}{8}T$$

Just in case generate these ratios.



$$\tan\alpha = \frac{v}{t_1} = 1.2$$
$$v = 1.2t_1$$
$$\tan\beta = \frac{v}{t_2} = 2$$
$$v = 2t_2$$
$$1.2t_1 = 2t_2$$
$$\frac{t_1}{t_2} = \frac{2}{1.2}$$

Find expressions for v in terms of T.

$$\tan\alpha = \frac{v}{t_1} = 1.2$$
$$v = 1.2t_1$$
$$v = 1.2\left(\frac{5}{8}T\right)$$
$$v = \frac{3}{4}T$$

Distance = Area Under Graph

$$696 = \frac{1}{2}T(v)$$
$$696 = \frac{1}{2}T\left(\frac{3}{4}T\right)$$
$$696 = \frac{3}{8}T^2$$
$$1856 = T^2$$
$$T = 8\sqrt{29} \text{ seconds}$$

In this type of question the 'acceleration' remains constant for the entire journey. We will be given information about various stages of a journey and need to solve for unknown variables, usually a and u.

The key is that the variables for both stages must represent the same number.

For example if we use u as the initial velocity from a to b it cannot also be used from b to c. The velocity has changed.

To overcome this we use u as the initial velocity and find distance equations for the sections a to b and then a to c. Keep measuring everything from a.

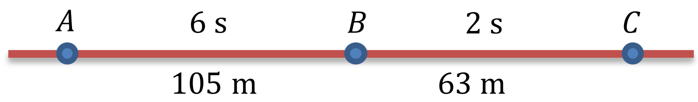
Another common question is when you are told that a particle travels a certain distance in a certain second.

The distance a particle travels in the 5th second is equal to the distance that the particle travels in 5 seconds minus the distance the particle travelled in 4 seconds.

Distance Travelled in 5th Second $= s_5 - s_4$.

We use this to set up simultaneous equations so that we can solve for u and a.

> A particle starts from rest and moves in a straight line with uniform acceleration. It passes three points a, b and c where $|ab| = 105$ m and $|bc| = 63$ m. If it takes 6 seconds to travel from a to b and 2 seconds to travel from b to c find its acceleration.



Use distance formula with A to B

$$s = ut + \frac{1}{2}at^2$$

$$105 = (u)(6) + \frac{1}{2}(a)(6)^2$$

$$105 = 6u + 18a$$

$$6u + 18a = 105$$

Use distance formula with A to C

$$s = ut + \frac{1}{2}at^2$$

$$168 = (u)(8) + \frac{1}{2}(a)(8)^2$$

$$168 = 8u + 32a$$

$$8u + 32a = 168$$

Then solve the simultaneous equation for u and a.

> A particle moving in a straight line with uniform acceleration describes 23 m in the fifth second of its motion and 31 m in the seventh second. Calculate its initial velocity.

$$S_5 - S_4 = 23$$

$$u(5) + \frac{1}{2}a(5)^2 - \left(u(4) + \frac{1}{2}a(4)^2\right) = 23$$

$$5u + 12.5a - 4u - 8a = 23$$

$$u + 4.5a = 23$$

$$S_7 - S_6 = 31$$

$$u(7) + \frac{1}{2}a(7)^2 - \left(u(6) + \frac{1}{2}a(6)^2\right) = 31$$

$$7u + 24.5a - 6u - 18a = 31$$

$$u + 6.5a = 31$$

Then solve the simultaneous equation for u and a.

$a = 4\ \text{m/s}^2$
$u = 5\ \text{m/s}$

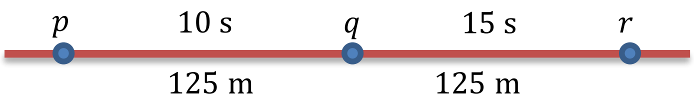
Revision Checklist

- 2015 (a) ☐
- 2010 (b) ☐
- 2003 (a) ☐
- 2002 (b) ☐
- 1996 (a) ☐
- 1995 (a) ☐
- 1993 (a) ☐
- 1988 (a) ☐
- 1986 (b) ☐
- 1985 ☐
- 1974 ☐

The points p, q and r all lie in a straight line. A train passes point p with speed u m/s.
The train is travelling with uniform retardation f m/s^2.
The train takes 10 seconds to travel from p to q and 15 seconds to travel from q to r, where $|pq| = |qr| = 125$ metres.

(i) Show that $f = \frac{1}{3}$
(ii) The train comes to rest s metres after passing r. Find s, giving your answer correct to the nearest metre.

(i)



The distance from p to q is 125 m.

$u = u$
$a = -f$
$s = 125$
$t = 10$

$$s = ut + \frac{1}{2}at^2$$
$$125 = (u)(10) + \frac{1}{2}(-f)(10)^2$$
$$125 = 10u - 50f$$

The distance from p to r is 250 m.
Note we find $|pr|$ rather than $|qr|$ so we can use same initial velocity u.

$u = u$
$a = -f$
$s = 250$
$(125 + 125)$
$t = 25$
$(10 + 15)$

$$s = ut + \frac{1}{2}at^2$$
$$250 = (u)(25) + \frac{1}{2}(-f)(25)^2$$
$$250 = 25u - 312.5f$$

Note:
f is the retardation so
$-f$ is the acceleration.

Solve the simultaneous equation

$$25u - 312.5f = 250$$
$$10u - 50f = 125 \quad \times -2.5$$

$$25u - 312.5f = 250$$
$$-25u + 125f = -312.5$$
$$-187.5 = -75.5$$
$$f = \frac{1}{3} \text{ m/s}^2$$

(ii)

Solve for u

$$10u - 50f = 125$$
$$10u - 50\left(\frac{1}{3}\right) = 125$$
$$10u - \frac{50}{3} = 125$$
$$10u = \frac{425}{3}$$
$$u = \frac{85}{6} \text{ m/s}$$

The train will have stopped when the final velocity is 0.

$u = \frac{85}{6}$
$v = 0$
$a = -\frac{1}{3}$
$s = s$

$$v^2 = u^2 + 2as$$
$$(0)^2 = \left(\frac{85}{6}\right)^2 + 2\left(-\frac{1}{3}\right)(s)$$
$$0 = \frac{7225}{36} - \frac{2}{3}s$$
$$\frac{2}{3}s = \frac{7225}{36}$$
$$s = \frac{7225}{24}$$
$$= 301\frac{1}{24} \text{ m}$$

This is the distance travelled before the train had stopped.

The question asks for how many metres after?

Subtract $301\frac{1}{24}$ from the distance travelled $|pr|$, 250m

$$= 301\frac{1}{24} - 250$$
$$= 51\frac{1}{24} \text{ m}$$

In this type of question we use the formula:

$F = ma$

$F =$ Net Force
$m =$ mass of the object
$a =$ acceleration of the object

$F = T - R$ (Object being pulled)
or $W - R$ (Falling due to gravity)
where
$T =$ Tractive (Pulling) Force
$W =$ Weight
$R =$ Resistance

Another useful formula to remember is Power

$P = Tv$

Power is measured in W so if given kW make sure to convert before using the formula.

Questions that have been asked:

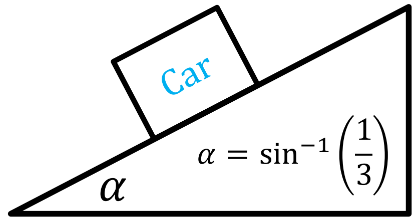
Car Towing Trailer Up Hill
2014, 2004, 1999

Mass Penetrating Soft Material
2005, 1982

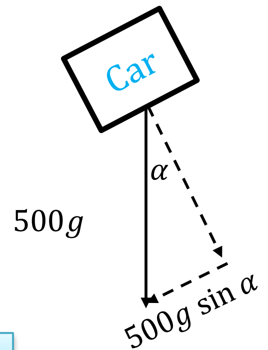
Bullet Penetrating Block of Wood
1970

For objects travelling up a hill we must resolve the Forces Parallel and Perpendicular to the incline.

A car of mass 500 kg moves up a hill. The force of the engine is 3000 N. There is air resistance of 100 N. Calculate the acceleration.



Resolve the downward force into components parallel and perpendicular to the incline.



$F = ma$

$3000 - 100 - 500g \sin \alpha = 500a$

$3000 - 100 - 500g\left(\frac{1}{3}\right) = 500a$

$a = \frac{38}{15}$ m/s^2

A particle of mass 3 grammes falls from rest from a height of 0.4 m on to a soft material into which it sinks 0.0245 m. Neglecting air resistance, calculate the constant resistance of the material.

Calculate the speed of the object at the moment in hits the soft material.

$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2(9.8)(0.4)$
$v^2 = 7.84$
$v = 2.8$ m/s

Calculate the acceleration of the object in the soft material given that it will stop when $v = 0$.

$v^2 = u^2 + 2as$
$0^2 = (2.8)^2 + 2a(0.0245)$
$0 = 7.84 + 0.049a$
$a = -160$ m/s^2

Calculate the resistance of the material.

$F = ma$

$W - R = 0.003(-160)$
$0.003(9.8) - R = 0.003(-160)$
$0.0294 - R = -0.48$
$R = 0.5094$ N

Revision Checklist

- 2014 (b) ☐
- 2005 (b) ☐
- 2004 (b) ☐
- 1999 (a) ☐
- 1994 (b) ☐
- 1982 (b) ☐
- 1970 ☐

A car of mass 1200 kg tows a caravan of mass 900 kg first along a horizontal road with acceleration f and then up an incline α to the horizontal road at uniform speed.
The force exerted by the engine is 2700 N. Friction and air resistance amount to 150 N on the car and 240 N on the caravan.
(i) Calculate the acceleration, f, of the car along the horizontal road.
(ii) Calculate the value of α, to the nearest degree.

(i)

First calculate the acceleration along horizontal road.

$F = ma$

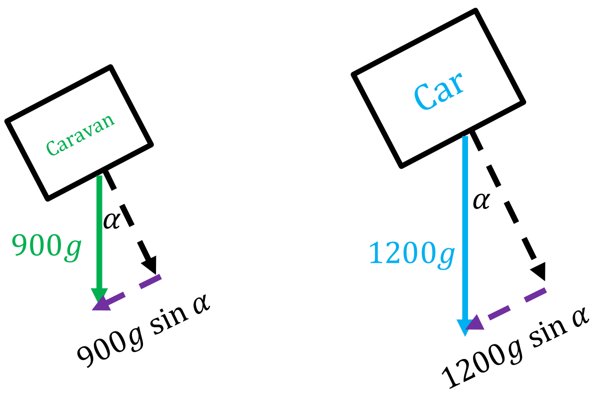
$2700 - 150 - 240 = 2100a$
$a = 1.1 \text{ m s}^{-2}$

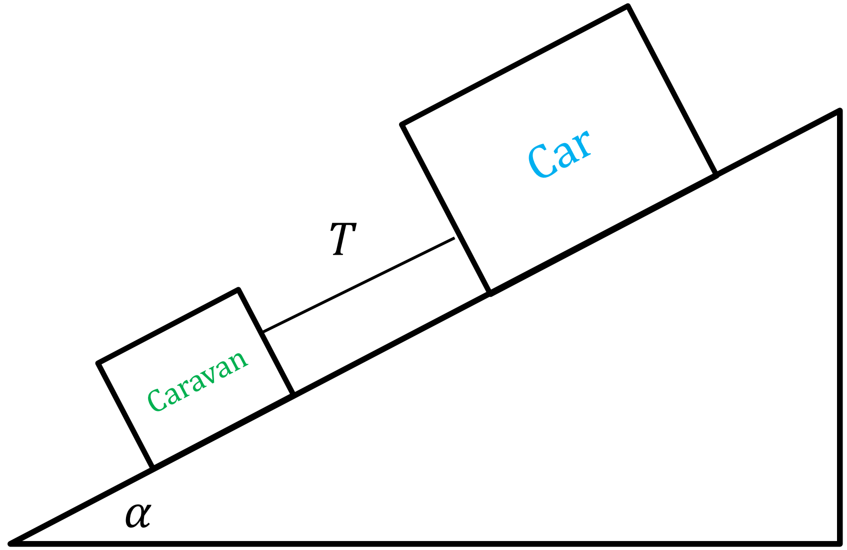
OR (…taking the Car and Caravan separately)

Car
$2700 - 150 - T$
Caravan
$T - 240$

Total
$2700 - 150 - T + T - 240 = 2100a$
$a = 1.1 \text{ m s}^{-2}$

Same Result.





Note: Uniform Speed means no acceleration!

(ii)

Up the incline there is uniform speed, therefore, acceleration is 0.

$F = ma$

Car
$2700 - 150 - T - 1200g\sin\alpha = 1200(0)$
$-T - 1200g\sin\alpha = -2550$
Caravan
$T - 900g\sin\alpha - 240 = 900(0)$
$T - 900g\sin\alpha = 240$

Solve the simultaneous equation.

$-T - 1200g\sin\alpha = -2550$
$T - 900g\sin\alpha = 240$
$-2310g\sin\alpha = -2550$
$$\sin\alpha = \frac{2550}{2310g}$$
$\alpha = 6.46766°$
$\alpha \approx 6°$