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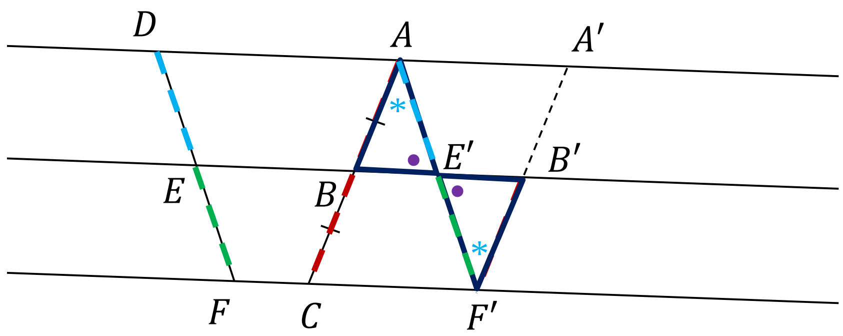
Junior & Leaving Cert

PROOFS, THEOREMS, DEFINITIONS

Leaving Cert Higher level

Diagram:



Given: $AD\|BE\|CF$, as in the diagram, with $|AB| = |BC|$

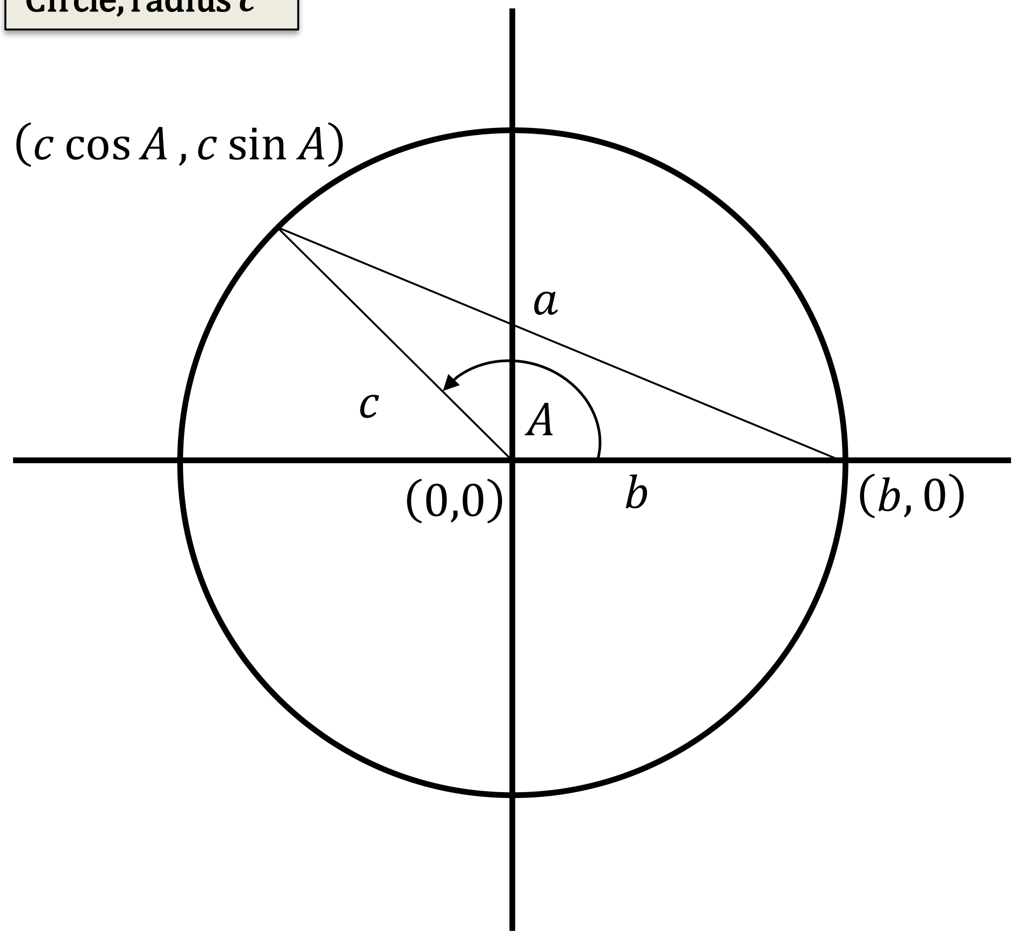
To Prove: $|DE| = |EF|$

Construction:
Draw $AE'||DE$, cutting EB at E' and CF at F'
Draw $F'B'||AB$, cutting EB at B', as in the diagram.

Proof:

$\|B'F'\| = \|BC\|$	opposite sides in a parallelogram, Theorem 9
$= \|AB\|$	by assumption
$\|\angle BAE'\| = \|\angle E'F'B'\|$	alternate angles
$\|\angle AE'B\| = \|\angle F'E'B'\|$	vertically opposite angles
$\therefore\ \Delta ABE'$ is congruent to $\Delta F'B'E'$	ASA
Therefore $\|AE'\| = \|F'E'\|$.	
But $\|AE'\| = \|DE\|$ and $\|F'E'\| = \|FE\|$	opposite sides in a parallelogram
$\therefore \|DE\| = \|EF\|$	

Circle, radius c



Distance Formula:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note:
$\cos^2 A + \sin^2 A = 1$

$$a = \sqrt{(c\cos A - b)^2 + (c\sin A - 0)^2}$$
$$a^2 = c^2\cos^2 A - 2bc\cos A + b^2 + c^2\sin^2 A$$
$$a^2 = c^2\cos^2 A - 2bc\cos A + b^2 + c^2\sin^2 A$$
$$a^2 = b^2 + c^2(\cos^2 A + \sin^2 A) - 2bc\cos A$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Proof that $\sqrt{2}$ is irrational, that it cannot be written $\frac{p}{q}$ where p and q are integers.

Suppose that $\sqrt{2}$ is rational, that it **can** be written as a fraction $\frac{p}{q}$ where p and q are integers with no common factors.

$$\left(\frac{p}{q}\right) = \sqrt{2}$$

square both sides

$$\left(\frac{p}{q}\right)^2 = 2$$

which implies

$$p^2 = 2q^2$$

The premise $p^2 = 2q^2$ tells us that p is even. Assuming p and q have no common factors, q must be odd. However the square of an even number is divisible by 4, which leads us to conclude that q is even. A contradiction.

Hence $\sqrt{2}$ cannot be written as a fraction $\frac{p}{q}$ where p and q are integers.