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Junior & Leaving Cert

ALGEBRA

Leaving Cert Ordinary level

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Algebra – Table of Contents

LCOL

Back

Show that $x = 4$ is a solution of the equation $x^2 - 2x - 8 = 0$.

To show that $x = 4$ is a solution substitute 4 for x and see if that value of x satisfies the equation (a TRUE statement).

$x^2 - 2x - 8 = 0$

$(4)^2 - 2(4) - 8 = 0$ Let $x = 4$

$16 - 8 - 8 = 0$

$0 = 0$

Which is TRUE, hence $x = 4$ is a solution to the equation $x^2 - 2x - 8 = 0$.

Alternate Method

Show that $x = 4$ is a solution of the equation $x^2 - 2x - 8 = 0$.

Alternate Method

Solve the equation through factorising

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

Factorise

$x - 4 = 0$ | $x + 2 = 0$

$x = 4$ | $x = -2$

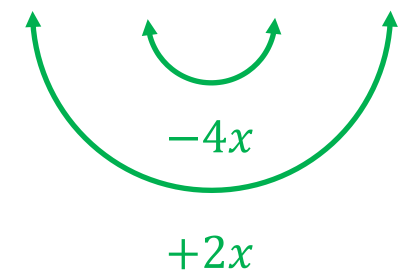
Let each bracket equal zero and solve for x

Hence, $x = 4$ is a solution to the equation $x^2 - 2x - 8 = 0$.

Trial and Error Approach

$x^2 - 2x - 8$

$(x - 4)(x + 2)$



4	
8	1
4	2

$-4x$

$+2x$

$-2x$

The equation $x^2 + ax + b = 0$, where $a, b \in \mathbb{Z}$, has solutions $x = 5$ and $x = -2$.

Find the value of a and the value of b.

Form a Quadratic Equation Given the Roots

$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

Roots

$x = 5, x = -2$

$$x^2 + (\text{sum of roots})x + \text{product of roots} = 0$$

$$x^2 - \big(5 + (-2)\big)x + (5)(-2) = 0$$

$$x^2 - 3x - 10 = 0$$

Alternate Method

$$x = 5, x = -2$$

$$(x - 5)(x + 2) = 0$$

$$x(x + 2) - 5(x + 2) = 0$$

$$x^2 + 2x - 5x - 10 = 0$$

$$x^2 - 3x - 10 = 0$$

$a = -3$
$b = -10$

Find the solutions of the equation $5x^2 - 2x - 9 = 0$ where $x \in \mathbb{R}$.

Give each answer correct to 2 decimal places.

$$5x^2 - 2x - 9 = 0$$

$a = 5$

$b = -2$

$c = -9$

$-b$ formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-9)}}{2(5)}$$

$$x = \frac{2 \pm \sqrt{4 + 180}}{10}$$

$$x = \frac{2 \pm \sqrt{184}}{10}$$

$$x = \frac{2 + \sqrt{184}}{10}$$

$$x = 1.56$$

$$x = \frac{-13 - \sqrt{193}}{4}$$

$$x = -1.16$$

Solve the equation:

$4(2x+3)-7=3(x-5)$, where $x \in \mathbb{R}$.

$$4(2x+3)-7=3(x-5)$$

$$8x+12-7=3x-15$$

$$8x-3x=7-15-12$$

$$5x=-20$$

$$x=\frac{-20}{5}$$

$$x=-4$$

A linear equation where we collect the x terms on one side of the equals and the numbers on the other.

Remove brackets first by multiplying the term outside the bracket by the terms inside.

Solve the simultaneous equations:

$$2x - y = 7$$

$$x^2 + y^2 = 49$$

Linear

Non Linear

Take the linear expression and express y in terms of x.

$$2x - y = 7$$
$$2x - 7 = y$$

Substitute this value of y into the non linear equation.

$$x^2 + y^2 = 49$$
$$x^2 + (2x - 7)^2 = 49$$
$$x^2 + (2x - 7)(2x - 7) = 49$$
$$x^2 + 2x(2x - 7) - 7(2x - 7) = 49$$
$$x^2 + 4x^2 - 14x - 14x + 49 = 49$$
$$5x^2 - 28x = 0$$
$$x(5x - 28) = 0$$

Factorise to solve the quadratic.

$$x = 0$$

$$5x - 28 = 0$$
$$5x = 28$$
$$x = \frac{28}{5}$$

We can now use our x values to solve for y by subbing them back into the rearranged linear equation.

$$y = 2x - 7$$
$$y = 2(0) - 7$$
$$y = 0 - 7$$
$$y = -7$$
$$(0, -7)$$

$$y = 2x - 7$$
$$y = 2\left(\frac{28}{5}\right) - 7$$
$$y = \frac{56}{5} - 7$$
$$y = \frac{21}{5}$$
$$\left(\frac{28}{5}, \frac{21}{5}\right)$$

Solve the equation:

$$\frac{9x-6}{2}=\frac{3x-14}{3}+\frac{9x}{4}$$

Note:
Numerator – top of fraction
Denominator – bottom of fraction

$$\frac{9x-6}{2}=\frac{3x-14}{3}+\frac{9x}{4}$$

$$\frac{6(9x-6)=4(3x-14)+3(9x)}{\not{12}}$$

Find common denominator.

Drop the denominator if there is an equals in the numerator.

$$54x-36=12x-56+27x$$

$$54x-12x-27x=36-56$$

$$15x=-20$$

$$x=-\frac{20}{15}$$

$$x=-\frac{4}{3}$$

Solve the simultaneous equations:

$$3x - y = 4$$
$$4x^2 - 3xy = 4.$$

Take the linear expression and express y in terms of x.

$$3x - y = 4$$
$$3x - 4 = y$$

Substitute this value of y into the non linear equation.

$$4x^2 - 3xy = 4$$
$$4x^2 - 3x(3x - 4) = 4$$
$$4x^2 - 9x^2 + 12x = 4$$
$$-5x^2 + 12x - 4 = 0$$
$$5x^2 - 12x + 4 = 0$$
$$(5x - 2)(x - 2) = 0$$

$$5x = 2 \qquad x = 2$$
$$x = \frac{2}{5}$$

We can now use our x values to solve for y by subbing them back into the rearranged linear equation.

$$y = 3x - 4$$
$$y = 3\left(\frac{2}{5}\right) - 4$$
$$y = \frac{6}{5} - 4$$
$$y = -\frac{14}{5}$$

$$\left(\frac{2}{5}, -\frac{14}{5}\right)$$

$$y = 3x - 4$$
$$y = 3(2) - 4$$
$$y = 6 - 4$$
$$y = 2$$

$$(2, 2)$$

Solve the equation

$$2^{9x-1} = 8^{2x}$$

POWERS

$$2^6$$

exponent

base

We need to try and change all terms so that they are of the same base number, in this case 2.

$$2^{9x-1} = 8^{2x}$$

$$2^{9x-1} = (2^3)^{2x}$$

$$2^{9x-1} = 2^{6x}$$

Note:
$8 = 2^3$

As the bases are equal so must be the exponents. Let them equal and solve for x.

$$9x - 1 = 6x$$

$$9x - 6x = 1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

Solve the equation $x^2 - 3x - 4 = 0$.

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$ | **Factorise**

$x - 4 = 0$ | $x + 1 = 0$

$x = 4$ | $x = -1$

Let each bracket equal zero and solve for x

Trial and Error Approach

$x^2 - 3x - 4$

$(x - 4)(x + 1)$

$-4x$

$+x$

4	
2	2
4	1

$-4x$

$+x$

$-3x$

Solve for x:

$$\frac{3x+1}{5}+\frac{x-2}{2}=\frac{47}{10}$$

Note:
Numerator – top of fraction
Denominator – bottom of fraction

$$\frac{3x+1}{5}+\frac{x-2}{2}=\frac{47}{10}$$

$$\frac{2(3x+1)+5(x-2)=1(47)}{\not{10}}$$

Find common denominator.

Drop the denominator if there is an equals in the numerator.

$$6x+2+5x-10=47$$

$$6x+5x=47+10-2$$

$$11x=55$$

$$x=\frac{55}{11}$$

$$x=5$$

Solve the simultaneous equations:

$$x - 5y = -13$$
$$x^2 + y^2 = 13$$

Take the linear expression and express x in terms of y.

$$x - 5y = -13$$
$$x = 5y - 13$$

Substitute this value of x into the non linear equation.

$$x^2 + y^2 = 13$$
$$(5y - 13)^2 + y^2 = 13$$
$$25y^2 - 130y + 169 + y^2 = 13$$
$$26y^2 - 130y + 156 = 0$$
$$y^2 - 5y + 6 = 0$$
$$(y - 3)(y - 2) = 0$$
$$y = 3 \qquad y = 2$$

We can now use our y values to solve for x by subbing them back into the rearranged linear equation.

$$x = 5y - 13$$
$$x = 5(3) - 13$$
$$x = 15 - 13$$
$$x = 2$$

$$(2, 3)$$

$$x = 5y - 13$$
$$x = 5(2) - 13$$
$$x = 10 - 13$$
$$x = -3$$

$$(-3, 2)$$

Solve the following inequality for $x \in \mathbb{R}$ **and** show your solution on the numberline below:

$$2(3 - x) < 8.$$

We treat inequalities like equations but need to be a little more careful when dividing by negative numbers.

Try and collect the x terms to the side where they will be positive.

If you end up with a negative x term and divide by that negative you must change the direction of the inequality.

Treat as if it were an equation collecting 'like' terms on the same side.

$2(3 - x) < 8$

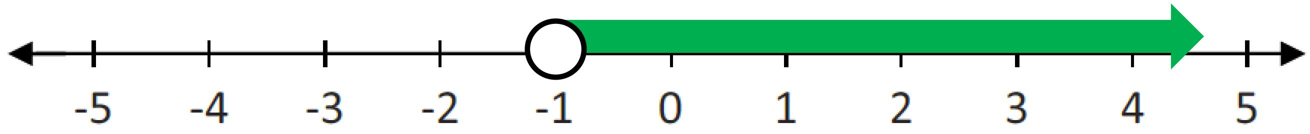
$6 - 2x < 8$

$6 - 8 < 2x$

$-2 < 2x$

$-1 < x$

The solution is the set of real numbers greater than -1.



$2(3 - x) < 8$

$6 - 2x < 8$

$-2x < 8 - 6$

$-2x < 2$

$x > -1$

$x \in \mathbb{R}$
$\mathbb{R}$ are the real numbers and include all the fractions and decimals.
These are represented by a shaded number line.
Full dots when equal to end numbers. Hollow dots when not equal.
Note the arrow to show that all numbers are included to infinity!

Solve for x:

$2^{2x-1} = 64.$

POWERS

2^6

exponent

base

We need to try and change all terms so that they are of the same base number, in this case 2.

$2^{2x-1} = 64$

$2^{2x-1} = 2^6$

Note:
$64 = 2^6$

As the bases are equal so must be the exponents.
Let them equal and solve for x.

$2x - 1 = 6$

$2x = 6 + 1$

$2x = 7$

$x = \frac{7}{2}$

Solve the equation $2x^2 - 7x - 3 = 0$. Give each answer correct to 2 decimal places.

$$2x^2 - 7x - 3 = 0$$

$a = 2$
$b = -7$
$c = -3$

$-b$ formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{49 + 24}}{4}$$

$$x = \frac{7 \pm \sqrt{73}}{4}$$

$$x = \frac{7 + \sqrt{73}}{4}$$

$$x = 3.89$$

$$x = \frac{7 - \sqrt{73}}{4}$$

$$x = -0.39$$

Solve the simultaneous equations below to find the value of a and the value of b.

$$2a + 3b = 15$$
$$5a + b = -8$$

① $2a + 3b = 15$
② $5a + b = -8$

Rewrite the equations so they are both in form (if not already)
$ax + by = c$
Label them ① and ②.

Then multiply one or both of the lines to get the x or y terms to cancel.

① $2a + 3b = 15$
② $5a + b = -8$ $\times -3$

① $2a + 3b = 15$
② $-15a - 3b = 24$

$-13a = 39$

$a = \frac{39}{-13}$

$a = -3$

Sub back into either original equation

② $5a + b = -8$
$5(-3) + b = -8$
$-15 + b = -8$
$b = 7$

$(-3, 7)$

Solve for x.

$$(x+5)(3x-4)-3(x^2+2)+4=0$$

$$(x+5)(3x-4)-3(x^2+2)+4=0$$

$$x(3x-4)+5(3x-4)-3x^2-6+4=0$$

$$\cancel{3x^2}-4x+15x-20-\cancel{3x^2}-6+4=0$$

$$-4x+15x=20+6-4$$

$$11x=22$$

$$x=\frac{22}{11}$$

$$x=2$$

Find the solutions of

$$\frac{5}{x+3} - \frac{1}{x} = \frac{1}{2}$$

where $x \neq -3, 0, x \in \mathbb{R}$.

Note:
Numerator – top of fraction
Denominator – bottom of fraction

$$\frac{5}{x+3} - \frac{1}{x} = \frac{1}{2}$$

$$\frac{5(2)(x) - 1(2)(x+3) = 1(x)(x+3)}{2(x)(x+3)}$$

The common denominator is the product of the denominators of each term. Multiply each numerator by the denominators of the other terms.
We can remove the denominator because there is an equals on top.

$$10x - 2x - 6 = x^2 + 3x$$

$$-x^2 + 10x - 2x - 3x - 6 = 0$$

$$-x^2 + 5x - 6 = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

Solve the quadratic equation by factorising or the $-b$ formula.

$x - 3 = 0$ $\qquad$ $x - 2 = 0$

$x = 3$ $\qquad$ $x = 2$

Find the two values of x for which $3x^2 - 6x - 8 = 0$.
Give each answer correct to 1 decimal place.

$$3x^2 - 6x - 8 = 0$$

$a = 3$
$b = -6$
$c = -8$

$-b$ formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-8)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 + 96}}{6}$$

$$x = \frac{6 \pm \sqrt{132}}{6}$$

$$x = \frac{6 + \sqrt{132}}{6}$$

$$x = 2.9$$

$$x = \frac{6 - \sqrt{132}}{6}$$

$$x = -0.9$$

Solve for x:

$$11x - 5(2x - 1) = 3(6 - x) + 3.$$

A linear equation where we collect the x terms on one side of the equals and the numbers on the other. Remove brackets first by multiplying the term outside the bracket by the terms inside.

$$11x - 5(2x - 1) = 3(6 - x) + 3$$

$$11x - 10x + 5 = 18 - 3x + 3$$

$$11x + 3x - 10x = 18 + 3 - 5$$

$$4x = 16$$

$$x = 4$$

Solve the simultaneous equations:

$$y + 5 = 2x$$
$$x^2 + y^2 = 25$$

Take the linear expression and express y in terms of x.

$$y + 5 = 2x$$
$$y = 2x - 5$$

Substitute this value of y into the non linear equation.

$$x^2 + y^2 = 25$$
$$x^2 + (2x - 5)^2 = 25$$
$$x^2 + 4x^2 - 20x + 25 = 25$$
$$5x^2 - 20x = 0$$
$$5x(x - 4) = 0$$
$$x = 0 \qquad x = 4$$

We can now use our x values to solve for y by subbing them back into the rearranged linear equation.

$$y = 2x - 5$$
$$y = 2(0) - 5$$
$$y = -5$$

$$(0, -5)$$

$$y = 2x - 5$$
$$y = 2(4) - 5$$
$$y = 8 - 5$$
$$y = 3$$

$$(4, 3)$$

Solve for $3(x-7)+5(x-4)=15$, where $x \in \mathbb{R}$.

$$3(x-7)+5(x-4)=15$$

$$3x-21+5x-20=15$$

Remove the brackets by multiplying and then collect the x terms on the left of the equals and the number terms on the right.

$$3x+5x=15+21+20$$

$$8x=56$$

$$x=\frac{56}{8}$$

Divide both sides by 8

$$x=7$$

It is not asked but no harm to verify (check) your answer by subbing $x=7$ back into the original.

$$3(x-7)+5(x-4)=15$$

$$3(7-7)+5(7-4)=15$$

$$3(0)+5(3)=15$$

$$0+15=15$$

True confirming that $x=7$

Solve the equations below to find the value of a and the value of b:

$$4a + 3b = -3$$
$$5a = 25 + 2b$$

① $4a + 3b = -3$
② $5a = 25 + 2b$

Rewrite the equations so they are both in form $ax + by = c$
Label them ① and ②.

Then multiply one or both of the lines to get the x or y terms to cancel.

① $4a + 3b = -3 \quad \times 2$
② $5a - 2b = 25 \quad \times 3$

① $8a + 6b = -6$
② $15a - 6b = 75$

$23a = 69$

$a = \frac{69}{23}$

$a = 3$

Sub back into either original equation

① $4a + 3b = -3$

$4(3) + 3b = -3$

$12 + 3b = -3$

$3b = -15$

$b = -5$

$(3, -5)$

List all the values of x that satisfy the inequality $2(2x - 3) + 6x < 25$, where $x \in \mathbb{N}$.

$x \in \mathbb{N}$
$\mathbb{N}$ are the natural numbers.
$\mathbb{N} = \{1, 2, 3, \ldots\}$

$$2(2x - 3) + 6x < 25$$

$$4x - 6 + 6x < 25$$

$$4x + 6x < 25 + 6$$

$$10x < 31$$

$$x < \frac{31}{10}$$

$$x < 3.1$$

We must list all the natural numbers that are less than 3.1

$= \{1, 2, 3\}$

Simplify $3(4-5x)-2(5-6x)$.

Remove the brackets by multiplying then simplify by collecting 'like' terms together.

$$3(4-5x)-2(5-6x)$$
$$=12-15x-10+12x$$
$$=2-3x$$

List all the values of x that satisfy the inequality $2 - 3x \geq -6, \;\; x \in N$.

$x \in \mathbb{N}$
$\mathbb{N}$ are the natural numbers.
$\mathbb{N} = \{1, 2, 3, \ldots\}$

Treat as if it were an equation collecting 'like' terms on the same side.

$$2 - 3x \geq -6$$
$$2 + 6 \geq 3x$$
$$8 \geq 3x$$
$$\frac{8}{3} \geq x$$

The solution is the set of Natural Numbers (positive whole numbers) less than $2\frac{2}{3}$

$x \in \{1,2\}$

$g(x)$ is a function and $(2 - 3x) \times g(x) = 15x^2 - 22x + 8$, for all $x \in R$.

Find $g(x)$.

Looks trickier than it is. Question is asking what multiplies by $2 - 3x$ to give $15x^2 - 22x + 8$.

Easiest method is to factorise $15x^2 - 22x + 8$.

$15x^2 - 22x + 8$

$(2 - 3x)(4 - 5x)$

$g(x) = 4 - 5x$

Alternatively, use long division.

$$
\begin{array}{r|l}
 & -5x + 4 \\
\hline
-3x + 2 & 15x^2 - 22x + 8 \\
 & 15x^2 - 12x \\
\hline
 & \quad -10x + 8 \\
 & \quad -10x + 8 \\
\hline
 & \qquad 0
\end{array}
$$

$g(x) = -5x + 4$

Solve the equation $-x^2 + 6x - 4 = 0$.

Give each solution correct to one decimal place.

$$-x^2 + 6x - 4 = 0$$

$a = -1$
$b = 6$
$c = -4$

Could also have changed all the signs $(x^2 - 6x + 4 = 0)$, in which case $a = 1, b = -6$ and $c = 4$.

$-b$ formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-4)}}{2(-1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 16}}{-2}$$

$$x = \frac{-6 \pm \sqrt{20}}{-2}$$

$$x = \frac{-6 + \sqrt{20}}{-2}$$

$$x = 5.2$$

$$x = \frac{-6 - \sqrt{20}}{-2}$$

$$x = 0.8$$

Solve for x:

$$2(4-3x)+12=7x-5(2x-7)$$

$$2(4-3x)+12=7x-5(2x-7)$$

$$8-6x+12=7x-10x+35$$

$$8-35+12=7x-10x+6x$$

$$-15=3x$$

$$\frac{-15}{3}=x$$

$$-5=x$$

A linear equation where we collect the x terms on one side of the equals and the numbers on the other. Remove brackets first by multiplying the term outside the bracket by the terms inside.

Verify your answer to (i) above.

Verifying an answer means checking it to make sure it is correct. Sub your solution back into the original equation and check that the left hand side (LHS) of the equation equals the right (RHS).

$$2(4-3x)+12=7x-5(2x-7)$$

$$2\big(4-3(-5)\big)+12=7(-5)-5(2(-5)-7)$$

$x=-5$, from (a) (i)

$$2(4+15)+12=7(-5)-5(2(-5)-7)$$

$$50=50$$

$LHS=RHS$ therefore the solution $x=-5$ is correct

Solve the simultaneous equations:

$$x + y = 7$$
$$x^2 + y^2 = 25$$

Take the linear expression and express x in terms of y.

$$x + y = 7$$
$$x = 7 - y$$

Substitute this value of x into the non linear equation.

$$x^2 + y^2 = 25$$
$$(7 - y)^2 + y^2 = 25$$
$$(7 - y)(7 - y) + y^2 = 25$$
$$7(7 - y) - y(7 - y) + y^2 = 25$$
$$49 - 7y - 7y + y^2 + y^2 = 25$$
$$2y^2 - 14y + 24 = 0$$
$$y^2 - 7y + 12 = 0$$
$$(y - 4)(y - 3) = 0$$
$$y = 4 \qquad y = 3$$

We can now use our y values to solve for x by subbing them back into the rearranged linear equation.

$$x = 7 - y$$
$$x = 7 - 4$$
$$x = 3$$

$$(3, 4)$$

$$x = 7 - y$$
$$x = 7 - 3$$
$$x = 4$$

$$(4, 3)$$

Solve the equation

$$x^2 - x - 6 = 0$$

This is a quadratic equation. The easiest way to solve is to factorise (if possible). Alternative method is to use the $-b$ formula.

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x - 3 = 0 \qquad x + 2 = 0$

$x = 3 \qquad x = -2$

Trial and Error Approach

$x^2 - x - 6$

$(x - 3)(x + 2)$

$-3x$

$+2x$

6	
6	1
3	2

$-3x$
$+2x$
$-x$

Show that $\frac{(a\sqrt{a})^3}{a^4}$ simplifies to $\sqrt{a}$.

$$\frac{(a\sqrt{a})^3}{a^4}$$

$$= \frac{\left(a\left(a^{\frac{1}{2}}\right)\right)^3}{a^4}$$

$$= \frac{\left(a^{\frac{3}{2}}\right)^3}{a^4}$$

$$= \frac{a^{\frac{9}{2}}}{a^4}$$

$$= a^{\frac{1}{2}}$$

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Solve the equation $49^x = 7^{2+x}$ and verify your answer.

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

When trying to solve for an x that is in the 'powers' of a question we try and express each term in the form k^n where k is some prime number, usually 2, 3, 5 or 7. By doing this we can let the indices equal and solve for x.

$$49^x = 7^{2+x}$$
$$(7^2)^x = 7^{2+x}$$
$$7^{2x} = 7^{2+x}$$

$$2x = 2 + x$$
$$2x - x = 2$$
$$x = 2$$

$$49^x = 7^{2+x}$$
$$49^2 = 7^{2+2}$$
$$49^2 = 7^4$$
$$2401 = 2401$$

Solve the equation $x^2 - 6x - 23 = 0$, giving your answer in the form $a \pm b\sqrt{2}$, where $a, b \in Z$.

$$x^2 - 6x - 23 = 0$$

$a = 1$
$b = -6$
$c = -23$

$-b$ formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-23)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 + 92}}{2}$$

$$x = \frac{6 \pm \sqrt{128}}{2}$$

$$x = \frac{6 \pm 8\sqrt{2}}{2}$$

$$x = 3 \pm 4\sqrt{2}$$

Solve the simultaneous equations

$$2r - s = 10$$
$$rs - s^2 = 12$$

Take the linear expression and express s in terms of r.

$$2r - s = 10$$
$$s = 2r - 10$$

Substitute this value of s into the non linear equation.

$$rs - s^2 = 12$$
$$r(2r - 10) - (2r - 10)^2 = 12$$
$$2r^2 - 10r - \big(2r(2r - 10) - 10(2r - 10)\big) = 12$$
$$2r^2 - 10r - (4r^2 - 20r - 20r + 100) = 12$$
$$2r^2 - 10r - 4r^2 + 20r + 20r - 100 = 12$$
$$-2r^2 + 30r - 112 = 0$$
$$r^2 - 15r + 56 = 0$$
$$(r - 8)(r - 7) = 0$$

$r - 8 = 0$ $\quad$ $r - 7 = 0$

$r = 8$ $\quad$ $r = 7$

We can now use our r values to solve for s by subbing them back into the rearranged linear equation.

$$s = 2r - 10$$
$$s = 2(8) - 10$$
$$s = 16 - 10$$
$$s = 6$$

(8,6)

$$s = 2(7) - 10$$
$$s = 14 - 10$$
$$s = 4$$

(7,4)

Solve the equation $27^{2x} = 3^{x+10}$.

When trying to solve for an x that is in the 'powers' of a question we try and express each term in the form k^n where k is some prime number, usually 2, 3, 5 or 7. By doing this we can let the indices equal and solve for x.

$27^{2x} = 3^{x+10}$

$(3^3)^{2x} = 3^{x+10}$

$3^{6x} = 3^{x+10}$

$6x = x + 10$

$5x = 10$

$x = 2$

Rules of Indices

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Verify that $3+\sqrt{2}$ is a root (solution) of the equation $x^2-6x+7=0$.

If $3+\sqrt{2}$ is a solution then when you sub it in for x and the left hand side (LHS) of the equation should equal 0 (the RHS).

$$x^2-6x+7=0$$

$$\left(3+\sqrt{2}\right)^2-6\left(3+\sqrt{2}\right)+7=0$$

$$3\left(3+\sqrt{2}\right)+\sqrt{2}\left(3+\sqrt{2}\right)-6\left(3+\sqrt{2}\right)+7=0$$

$$9+3\sqrt{2}+3\sqrt{2}+2-18-6\sqrt{2}+7=0$$

$$6\sqrt{2}-6\sqrt{2}+18-18=0$$

$$0=0$$

Which is true.

Verify that $3+\sqrt{2}$ is a root (solution) of the equation $x^2-6x+7=0$.

Alternate Method

$$x^2-6x+7=0$$

Solve the quadratic equation using the $-b$ formula.

$a=1$
$b=-6$
$c=7$

$-b$ formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(7)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{36-28}}{2}$$

$$x=\frac{6\pm\sqrt{8}}{2}$$

$$x=\frac{6+2\sqrt{2}}{2}$$
$$x=3+\sqrt{2}$$

$$x=\frac{6-2\sqrt{2}}{2}$$
$$x=3-\sqrt{2}$$

Hence $x=3+\sqrt{2}$ is a solution to the equation.

Solve the equation $\frac{1}{2}(7x - 2) + 5 = 2x + 7$.

$$\frac{1}{2}(7x - 2) + 5 = 2x + 7$$

$$7x - 2 + 10 = 4x + 14$$

$$7x - 4x = 14 - 10 + 2$$

$$3x = 6$$

$$x = 2$$

A linear equation where we collect the x terms on one side of the equals and the numbers on the other. Remove brackets first by multiplying the term outside the bracket by the terms inside.

Solve the equation $\frac{2}{3x-4} - \frac{1}{2x+1} = \frac{1}{2}$ and give your answers correct to one decimal place.

The common denominator is the product of the denominators of each term. Multiply each numerator by the denominators of the other terms.

We can remove the denominator because there is an equals on top.

$$\frac{2}{3x-4} - \frac{1}{2x+1} = \frac{1}{2}$$

$$\frac{2(2)(2x+1) - 1(2)(3x-4) = 1(3x-4)(2x+1)}{2(3x-4)(2x+1)}$$

$$4(2x+1) - 2(3x-4) = 3x(2x+1) - 4(2x+1)$$

$$8x + 4 - 6x + 8 = 6x^2 + 3x - 8x - 4$$

$$6x^2 - 7x - 16 = 0$$

Solve the quadratic equation using the $-b$ formula.

$a = 6$

$b = -7$

$c = -16$

$-b$ formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-16)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 384}}{12}$$

$$x = \frac{7 \pm \sqrt{433}}{12}$$

$$x = \frac{7 + \sqrt{433}}{12}$$

$$x = 2.3$$

$$x = \frac{7 - \sqrt{433}}{12}$$

$$x = -1.2$$

Solve the simultaneous equations:

$$2f + \frac{2}{3}g + 1 = 0$$
$$f + \frac{1}{2}g + 1 = 0.$$

Rewrite the equations so they are both in form (if not already) $ax + by = c$
Label them ① and ②.

Then multiply one or both of the lines to get the x or y terms to cancel.

① $2f + \frac{2}{3}g + 1 = 0 \quad \times 3 \longrightarrow 6f + 2g + 3 = 0$

② $f + \frac{1}{2}g + 1 = 0. \quad \times 2 \longrightarrow 2f + g = -2$

① $6f + 2g = -3$
② $2f + g = -2 \quad \times -2$

① $6f + 2g = -3$
② $-4f - 2g = 4$

$2f = 1$

$f = \frac{1}{2}$

Sub back into either equation

① $6f + 2g = -3$

$6\left(\frac{1}{2}\right) + 2g = -3$

$3 + 2g = -3$

$2g = -6$

$g = -3$

Solve the following inequality, and show the solution set on the number line below.

$$5 - \frac{3}{4}x \leq \frac{19}{8}$$

Assuming $x \in \mathbb{R}$ as not stated in question (unusually).
$\mathbb{R}$ are the real numbers.
This includes all fractions and decimals and so we represent this on the number line with a shaded line.

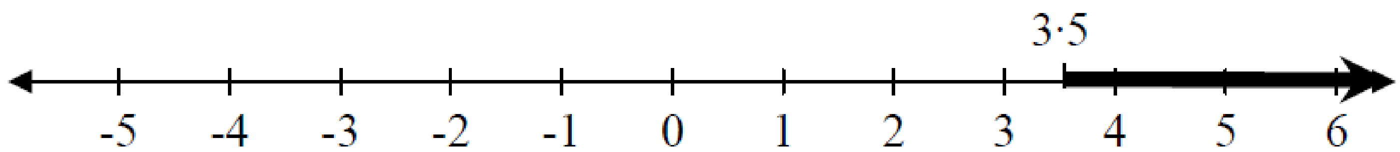
$$5 - \frac{3}{4}x \leq \frac{19}{8}$$

$$5 - \frac{19}{8} \leq \frac{3}{4}x$$

$$\frac{21}{8} \leq \frac{3}{4}x$$

$$\frac{\frac{21}{8}}{\frac{3}{4}} \leq x$$

$$3.5 \leq x$$



The solution set is all of the Real numbers ($\mathbb{R}$) greater than or equal to 3.5.