# Complex Numbers

## Leaving Cert Higher Level

### 2021 LCHL Paper 1 – Question 1 (a)

$\frac{(4-2i)}{(2+4i)} = 0 + ki$, where $k \in \mathbb{Z}$, and $i^2 = -1$. Find the value of $k$.

### 2021 LCHL Paper 1 – Question 1 (b)

Find $\sqrt{-5+12i}$.

Give both your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

### 2021 LCHL Paper 1 – Question 1 (c)

Use De Moivre's theorem to find the **three** roots of $z^3 = -8$.

Give each of your answers in the form $a + bi$, where $a, b \in \mathbb{R}$, and $i^2 = -1$.

### 2020 LCHL Paper 1 – Question 2 (a)

Find the two complex numbers $z_1$ and $z_2$ that satisfy the following simultaneous equations, where $i^2 = -1$:

$iz_1 = -4 + 3i$

$3z_1 - z_2 = 11 + 17i$

Write your answers in the form $a + bi$ where $a, b \in \mathbb{Z}$.

### 2020 LCHL Paper 1 – Question 2 (b)

The complex numbers $3 + 2i$ and $5 - i$ are the first two terms of a **geometric** sequence.

(i) Find $r$, the common ratio of the sequence.

Write your answer in the form $a + bi$ where $a, b \in \mathbb{Z}$.

(ii) Use de Moivre's Theorem to find $T_9$, the ninth term of the sequence.

Write your answer in the form form $a + bi$ where $a, b \in \mathbb{Z}$.

## 2019 LCHL Paper 1 – Question 5 (a)

$3 + 2i$ is a root of $z^2 + pz + q = 0$, where $p, q \in \mathbb{R}$, and $i^2 = -1$.

Find the value of $p$ and the value of $q$.

## 2019 LCHL Paper 1 – Question 5 (b)

(i) $v = 2 - 2\sqrt{3}i$. Write $v$ in the form $r(\cos\theta + i\sin\theta)$, where $r \in \mathbb{R}$ and $0 \leq \theta \leq 2\pi$.

(ii) Use your answer to part (b)(i) to find the two possible values of $w$, where $w^2 = v$.

Give your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

## 2018 LCHL Paper 1 – Question 4 (a)

Prove, using induction, that if $n$ is a positive integer then

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, where $i^2 = -1$.

## 2018 LCHL Paper 1 – Question 4 (b)

Hence, or otherwise, find $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3$ in its simplest form.

## 2017 LCHL Paper 1 – Question 2

$z = -\sqrt{3} + i$, where $i^2 = -1$.

(a) Use De Moivre's Theorem to write in the form $z^4$, where $a + b\sqrt{c}i$, where $a, b$, and $c \in \mathbb{Z}$.

(b) The complex number $w$ is such that $|w| = 3$ and $w$ makes an angle of $30°$ with the positive sense of the real axis. If $t = zw$, write $t$ in its simplest form.

## 2016 LCHL Paper 1 – Question 1 (a)

$(-4 + 3i)$ is one root of the equation $az^2 + bz + c = 0$, where $a,\ b,\ c \in \mathbb{R}$, and $i^2 = -1$.

Write the other root.

### 2016 LCHL Paper 1 – Question 1 (b)

Use De Moivre's Theorem to express $(1+i)^8$ in its simplest form.

### 2016 LCHL Paper 1 – Question 1 (c)

$(1+i)$ is a root of the equation $z^2+(-2+i)z+3-i=0$.

Find its other root in the form $m+ni$, where $m,\ n \in \mathbb{R}$, and $i^2=-1$.

### 2015 LCHL Paper 1 – Question 4 (a)

The complex numbers $z_1$, $z_2$ and $z_3$ are such that $\frac{2}{z_1}=\frac{1}{z_2}+\frac{1}{z_3}$, $z_2=2+3i$ and $z_3=3-2i$ where $i^2=-1$.

Write $z_1$ in the form $a+bi$, where $a, b \in Z$.

### 2015 LCHL Paper 1 – Question 4 (b)

Let $\omega$ be a complex number such that $\omega^n=1$, $\omega \neq 1$, and $S=1+\omega+\omega^2+\cdots+\omega^{n-1}$.

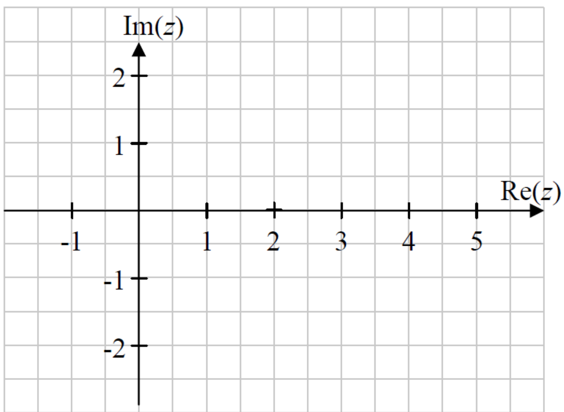
Use the formula for the sum of a finite geometric series to write the value of $S$ in its simplest form.

(In the Complex Numbers section but a knowledge of Sequences and Series more important.)

### 2014 LCHL Paper 1 – Question 2

Let $z_1=1-2i$, where $i^2=-1$.

The complex number $z_1$ is a root of the equation $2z^3-7z^2+16z-15$




(a) Find the other two roots of the equation.

(b) Let $w=z_1\overline{z_1}$, where $\overline{z_1}$ is the conjugate of $z_1$. Plot $z_1$, $\overline{z_1}$ and $w$ on the Argand diagram and label each point.

(c) Find the measure of the acute angle, $\overline{z_1}wz_1$, formed by joining $\overline{z_1}$ to $w$ to $z_1$ on the diagram. Give your answer correct to the nearest degree.

### 2014 LCHL Sample Paper 1 – Question 1 (a)

$w = -1 + \sqrt{3}i$ is a complex number, where $i^2 = -1$.

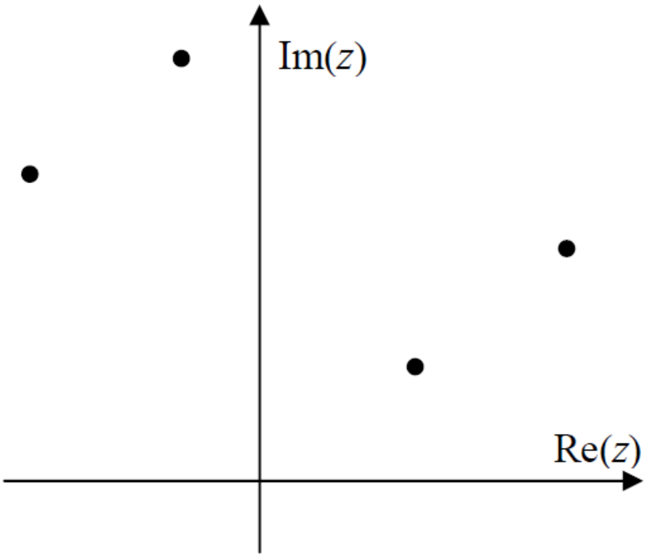
(i) Write $w$ in polar form.

(ii) Use De Moivre's theorem to solve the equation $z^2 = -1 + \sqrt{3}$.

Give your answer(s) in rectangular form.

### 2014 LCHL Sample Paper 1 – Question 1 (b)

Four complex numbers $z_1$, $z_2$, $z_3$ and $z_4$ are shown on the Argand diagram. They satisfy the following conditions:




$z_2 = iz_1$

$z_3 = kz_1$ where $k \in \mathbb{R}$

$z_4 = z_2 + z_3$

The same scale is used on both axes.

(a) Identify which number is which, by labelling the points on the diagram.

(b) Write down the approximate value of $k$

### 2013 LCHL Paper 1 – Question 1

$z = \frac{4}{1+\sqrt{3}i}$ is a complex number, where $i^2 = -1$

(a) Verify that $z$ can be written as $1 - \sqrt{3}i$.

(b) Plot $z$ on an Argand diagram and write $z$ in polar form.

(c) Use De Moivre's theorem to show that $z^{10} = -2^9\left(1 - \sqrt{3}i\right)$

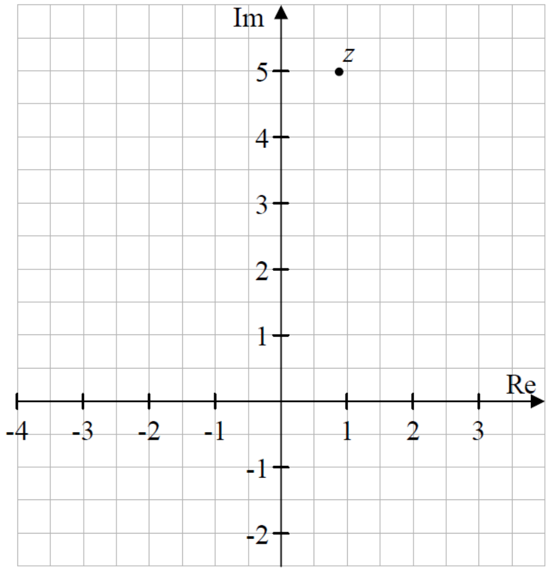
### 2012 LCHL Paper 1 – Question 3

The complex number $z$ has modulus $5\frac{1}{16}$ and argument $\frac{4\pi}{9}$.

(a) Find, in polar form, the four complex fourth roots of $z$. (That is, find the four values of $w$ for which $w^4 = z$.)

$z$ is marked on the Argand diagram below.

(b) On the same diagram, show the four answers to part (a).




### 2011 LCHL Paper 1 – Question 2 (a)

(i) Write the complex number $1 - i$ in polar form.

(ii) Use De Moivre's Theorem to evaluate $(1 - i)^9$, giving your answer in the rectangular form.

### 2011 LCHL Paper 1 – Question 2 (b)

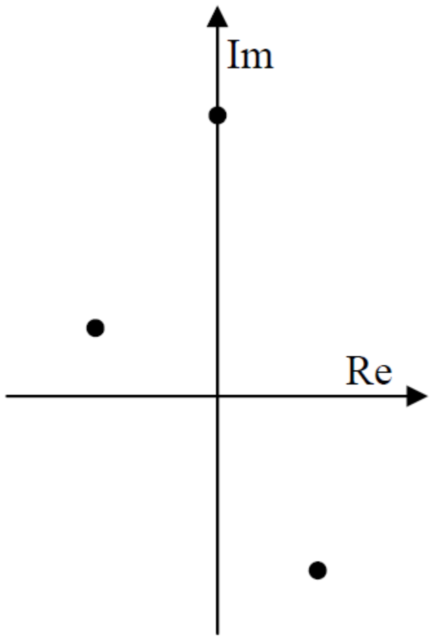
A complex number $z$ has modulus greater than 1.

The three numbers $z$, $z^2$, and $z^3$ are shown on the Argand diagram.

One of them lies on the imaginary axis, as shown.

(i) Label the points on the diagram to show which point corresponds to which number.

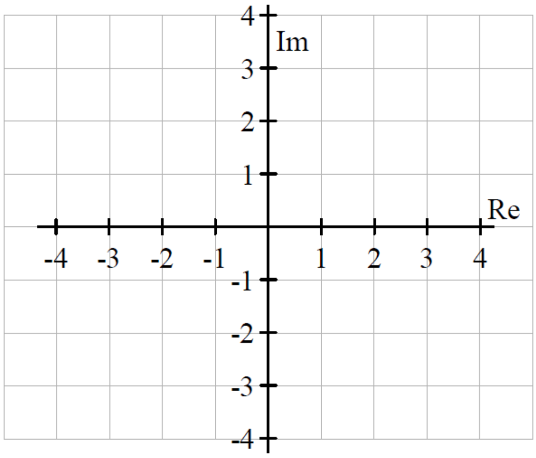
(ii) Find $\theta$, the argument of $z$.



## Relevant Leaving Cert Ordinary Level

### 2011 LCOL Sample Paper 1 – Question 5

$z$ is the complex number $1 + i$, where $i^2 = -1$.




(a)

(i) Find $z^2$ and $z^3$.

(ii) Verify that $z^4 = -4$

(iii) Show $z, z^2, z^3$ and $z^4$ on the Argand diagram.

(iv) Make one observation about the pattern of points on the diagram.

(b) Using the value of $z^4$, or otherwise, find the values of $z^8$, $z^{12}$ and $z^{16}$, and insert their values in the table below.

(c) Based on the pattern of values in part **(b)**, or otherwise, state whether $z^{40}$ is positive or negative. Explain how you got your answer.

(d) Write $z^{40}$ as a power of 2.

(e) Find $z^{41}$.

(f) On an Argand diagram, how far from the origin is $z^{41}$.

**For more Complex Numbers questions see the Complex Numbers EXTRA Pack.**