MATHSPOINTS.IE
JUNIOR & LEAVING CERT

COORDINATE GEOMETRY OF THE LINE

LEAVING CERT ORDINARY LEVEL

Points

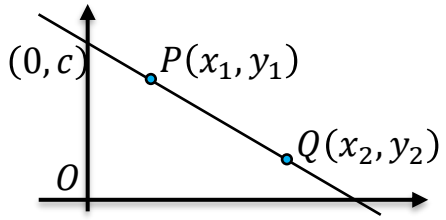
If we have **two points** (x_1, y_1) and (x_2, y_2) we can use formulae in the tables to find:

Distance
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope
$\frac{y_2 - y_1}{x_2 - x_1}$

Equation of a line
$y - y_1 = m(x - x_1)$



All formulae here relevant when answering Circle questions.

Line in the form $ax + by + c = 0$

We can find:
- where it **crosses the x axis** by letting $y = 0$
- where it **crosses the y axis** by letting $x = 0$

(also do this to **draw a line**)

- the **slope m,** using $-\frac{a}{b}$
- if a **point is on the line** by subbing the values of the point (x_1, y_1) in for x and y.

Slopes

Note the 4 ways to find slopes.

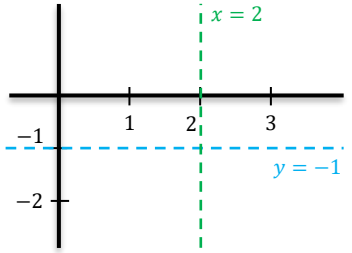
1. If we have two points use $\frac{y_2 - y_1}{x_2 - x_1}$
2. If we have a line in format $y = mx + c$ the slope is number beside the x
3. If we have a line in the format $ax + by + c = 0$ use $-\frac{a}{b}$
4. If we have a graph with reliable points we can use $\frac{rise}{run}$

Perpendicular Slope
Turn the slope upside down and change the sign.
If a line has a slope of $\frac{3}{5}$ the perpendicular slope is $-\frac{5}{3}$
To prove slopes perpendicular $m_1 m_2 = -1$

Parallel Slope
If the lines are parallel then the slopes are equal

Parallel Lines
If $ax + by + c = 0$ is a line then a **parallel** line can be written $ax + by + k = 0$
A perpendicular line can be written $bx - ay + k = 0$

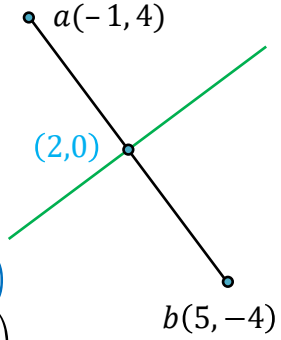
Lines Parallel to the Axes
$x = 2$ is a line parallel to the y-axis through 2 on the x axis
$y = -1$ is a line parallel to the x-axis through -1 on the y axis



Combination of Methods

$a(-1, 4)$ and $b(5, -4)$ are two points. Find the equation of the perpendicular bisector (line through midpoint) of $[ab]$.

$\frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-4 - 4}{5 - (-1)}$
$= \frac{-8}{6} = -\frac{4}{3}$
$-\frac{4}{3} \perp \frac{3}{4}$



$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= \left(\frac{-1 + 5}{2}, \frac{4 - 4}{2}\right)$
$= (2,0)$

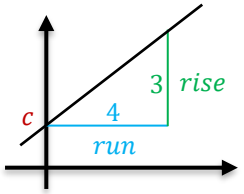
$y - y_1 = m(x - x_1)$
$y - 0 = \frac{3}{4}(x - 2)$
$4y = 3x - 6$
$\mathbf{3x - 4y - 6 = 0}$

Where does this line cut the y-axis
$3x - 4y - 6 = 0$
$3(0) - 4y - 6 = 0$
$4y = -6$
$y = -1.5 \qquad (0, -1.5)$

Line in the form $y = mx + c$

m will be the slope

$m = \frac{rise}{run} = \frac{3}{4}$



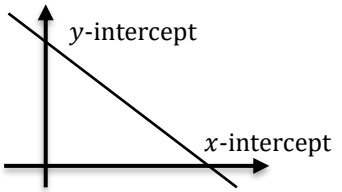
c the y-intercept
(the place where the line crosses y axis)

Slope and y-intercept

$x = 3 - y \longrightarrow y = -1x + 3$
$2x - 4y = 3 \longrightarrow y = \frac{1}{2}x - \frac{3}{4}$
$y = -\frac{1}{4}(2x - 7) \longrightarrow y = -\frac{1}{2}x + \frac{7}{4}$
$4x - 2y - 5 = 0 \longrightarrow y = 2x - \frac{5}{2}$
$x + \sqrt{3}y - 10 = 0 \longrightarrow y = -\frac{1}{\sqrt{3}}x + 10$

Graphing Lines

To draw a line two points are needed. The easiest points to find are where the lines cross axis.
Let $y = 0$ and find x
Let $x = 0$ and find y
Plot these two points
Draw the line through these points



Area of a Triangle

If given the 3 vertices of the triangle we can use the formula in the tables. We move one of the points to (0,0) and the others the same distance through translation.

Area of a triangle $\frac{1}{2}|x_1y_2 - x_2y_1|$

Or you can use $\frac{1}{2}$ base $\times$ height if we somehow find the base and height.

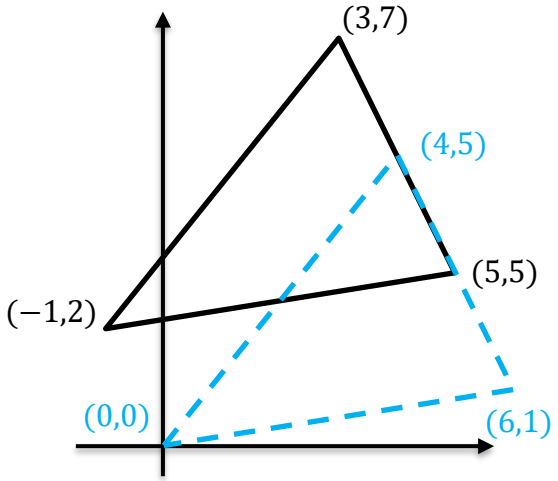
The modulus symbol | | means that we take the positive value of the answer.

Find the area of the triangle with vertices $p(-1,2)$ $q(5,3)$ $s(3,7)$

One of the points MUST be (0,0)

Translate
$p(-1,2) \rightarrow (0,0)$
$q(5,3) \rightarrow (6,1)$
$s(3,7) \rightarrow (4,5)$

$\frac{1}{2}|x_1y_2 - x_2y_1|$
$= \frac{1}{2}|(6)(5) - (1)(4)|$
$= \frac{1}{2}|30 - 40|$
$= \frac{1}{2}|26|$
$= 13$ square units



Remember

For coordinate geometry questions:

- Open the relevant **formula pages in the tables**.
- Always **draw a rough sketch**.
- Remember to sketch a line you must find two points and the easiest to find are where $x = 0$ and $y = 0$
- When given the equation of a line always write down its slope. If the slope is positive the line goes up. If the slope is negative the line goes down.
- Know also how to sketch a line given its slope (rise over run!).
- If you need a bit from one question to do the next but don't have it, make your best guess and move forward. Always show the examiner what you CAN do.

Point of Intersection of Two Lines

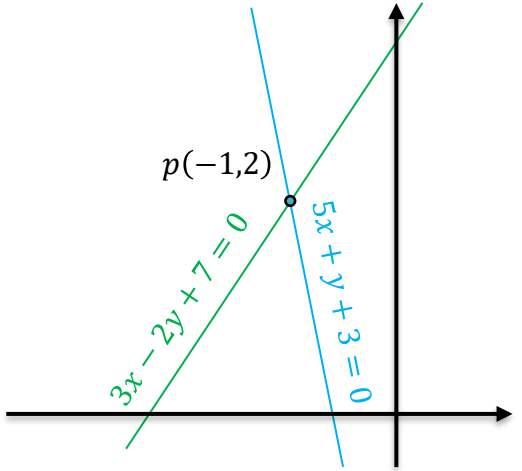
To find the point of intersection of two lines we do a simultaneous equation.

The line $L_1: 3x - 2y + 7 = 0$ and the line $L_2: 5x + y + 3 = 0$ intersect at the point p. Find p.

$3x - 2y + 7 = 0$ ①
$5x + y + 3 = 0$ ②

$3x - 2y = -7$ ①
$10x + 2y = -6$ ② $\times 2$
$13x = -13$
$x = -1$

$3x - 2y = -7$
$3(-1) - 2y = -7$
$4 = 2y$
$y = 2$
$(-1,2)$



Transformations of the Plane

- Translation: A translation moves a point a given distance and direction.
- Central Symmetry: Is a reflection in a point
- Axial Symmetry: Is a reflection in a line
- Axial Symmetry in the axes or central symmetry in the origin
  - In the x-axis – change the sign of y
  - In the y- axis – change the sign of x
  - Central symmetry – change both signs

A given rule

Find the image of the point (3, 1) through the translation $(2,-1) \rightarrow (4,1)$

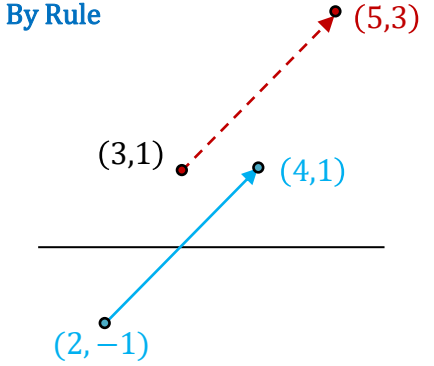
$(x: \uparrow 2, y: \downarrow 2)$

$(3,1) \rightarrow (5,3)$ under the above translation

Symmetry

An example would be to find image of the point (3, 1) through central symmetry in (1, 2).
$(3,1) \rightarrow (1,2) \rightarrow (-1,3)$

By Rule



Symmetry