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Junior & Leaving Cert

FORMAL GEOMETRY PROOFS

Leaving Cert Higher level

Formal Proofs of Theorems

Leaving Cert Theorems

Theorem 11
If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

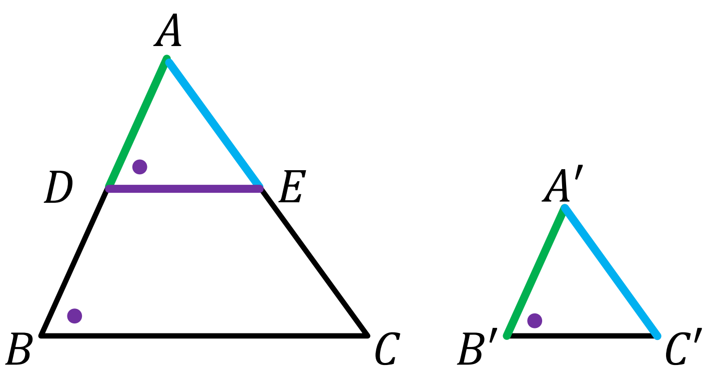
Theorem 12
Let ΔABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio.

Theorem 13
If two triangles ΔABC and ΔA' B' C' are similar, then their sides are proportional, in order.

Syllabus Wording:

Students will study all the theorems and corollaries prescribed for LCOL, but will not, in general, be asked to reproduce their proofs in examination.
However, they may be asked to give proofs of the Theorems 11, 12, 13, concerning ratios, which lay the proper foundation for the proof of Pythagoras studied at JC.

Diagram:



Given:
The similar triangles ΔABC and $\Delta A'B'C'$

To Prove:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$$

Construction:
We may suppose $|A'B'| \leq |AB|$.
Mark D on a $[AB]$ such that $[AD] = [A'B']$
Mark E on a [AC] with $[AE] = [A'C']$
Join D to E

Proof:
ΔADE is congruent to $\Delta A'B'C'$ — SAS
$\therefore |\angle ADE| = |\angle ABC|$ •
$\therefore DE \parallel BC$ • — Corresponding angles

$$\therefore \frac{|AB|}{|AD|} = \frac{|AC|}{|AE|}$$ — Theorem 12

$$\therefore \frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|}$$

Similarly,

$$\frac{|BC|}{|B'C'|} = \frac{|AB|}{|A'B'|}$$

$$\therefore \frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$$

Note that in the 2014 Marking Scheme A'' was used in place of D and C'' was used in place of E. Hopefully this makes for easier reading!

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TRIGONOMETRIC PROOFS

Leaving Cert Higher level

Trigonometric Identities

Must Know the Formal Proofs of:

1. $\cos^2 A + \sin^2 A = 1$

2. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

3. $a^2 = b^2 + c^2 - 2bc\cos A$

4. $\cos(A - B) = \cos A\cos B + \sin A\sin B$

5. $\cos(A + B) = \cos A\cos B - \sin A\sin B$

6. $\cos 2A = \cos^2 A - \sin^2 A$

7. $\sin(A + B) = \sin A\cos B + \cos A\sin B$

9. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

Formal Proofs Not Required For:

8. $\sin(A - B) = \sin A\cos B - \cos A\sin B$

9. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

10. $\sin 2A = \frac{2\tan A}{1 + \tan^2 A}$

11. $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

12. $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

13. $2\cos A\cos B = \cos(A + B) + \cos(A - B)$

14. $2\sin A\cos B = \sin(A + B) + \sin(A - B)$

15. $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

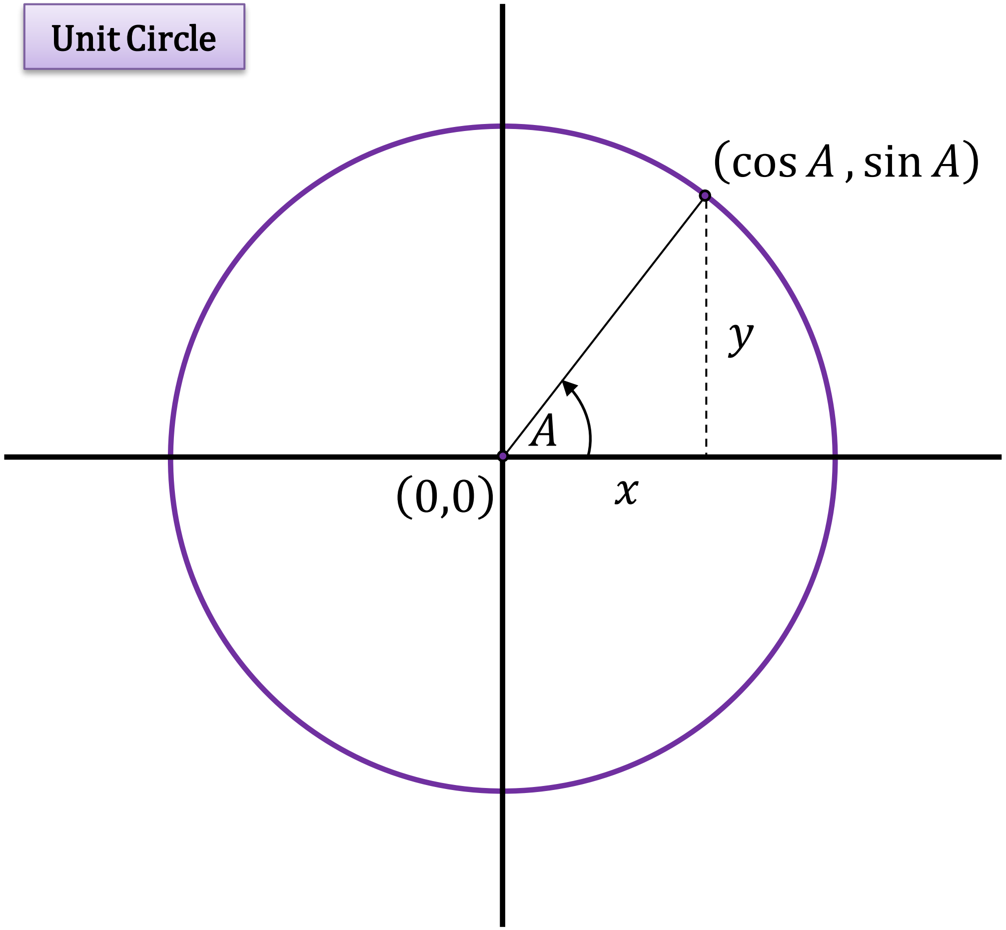
16. $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

17. $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

18. $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

It will be assumed that these formulae are established in the order listed here. In deriving any formula, use may be made of formulae that precede it.

Unit Circle



The distance from (0,0) to $(\cos A , \sin A)$ is 1.

Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(\cos A - 0)^2 + (\sin A - 0)^2} = 1$$

$$\cos^2 A + \sin^2 A = 1$$

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Junior & Leaving Cert

PAPER 1: FORMAL PROOFS AND CONSTRUCTIONS

Leaving Cert Higher level

Paper 1 Formal Proofs and Constructions

Formal Proofs

Proof By Contradiction – $\sqrt{2}$ is not Rational

Sum of Geometric Series by Induction

Sum to Infinity of Geometric Series (Limits)

Derive Amortisation Formula

De Moivre's Theorem by Induction

Paper 1 Construction

Construction of $\sqrt{2}$

Construction of $\sqrt{3}$

Other Things to Learn Off

De Moivre to Prove Trigonometric Identity I

De Moivre to Prove Trigonometric Identity II

Differentiate by 1st Principles

Proof by Contradiction is where we cannot directly prove a statement but we can prove that the opposite statement is false.

Suppose that $\sqrt{2}$ is rational, that it **can** be written as a fraction $\frac{p}{q}$ where p and q are integers with no common factors.

$$\frac{p}{q} = \sqrt{2}$$

$$\Rightarrow \left(\frac{p}{q}\right)^2 = 2$$

$$\Rightarrow p^2 = 2q^2$$

$\Rightarrow p^2$ is even

$\Rightarrow p$ is even

Let $p = 2k$ for some $k \in \mathbb{Z}$

$$p^2 = 2q^2$$

$$(2k)^2 = 2q^2$$

$$4k^2 = 2q^2$$

$$2k^2 = q^2$$

$\Rightarrow q^2$ is even

$\Rightarrow q$ is even

Let $q = 2m$ for some $m \in \mathbb{Z}$

$$\therefore \sqrt{2} = \frac{p}{q} = \frac{2k}{2m}$$

$\Rightarrow$ Common factor of 2, which is a contradiction.

Hence $\sqrt{2}$ cannot be written as a fraction $\frac{p}{q}$ where p and q are integers with no common factors.