

► 2023 LCHL Paper 1 – Question 7

Fiona is driving on a motorway. She passes a point **A** on the motorway. Her speed is given by:

$$v(t) = \frac{2}{3}t^3 - 6t^2 + 13t + 109$$

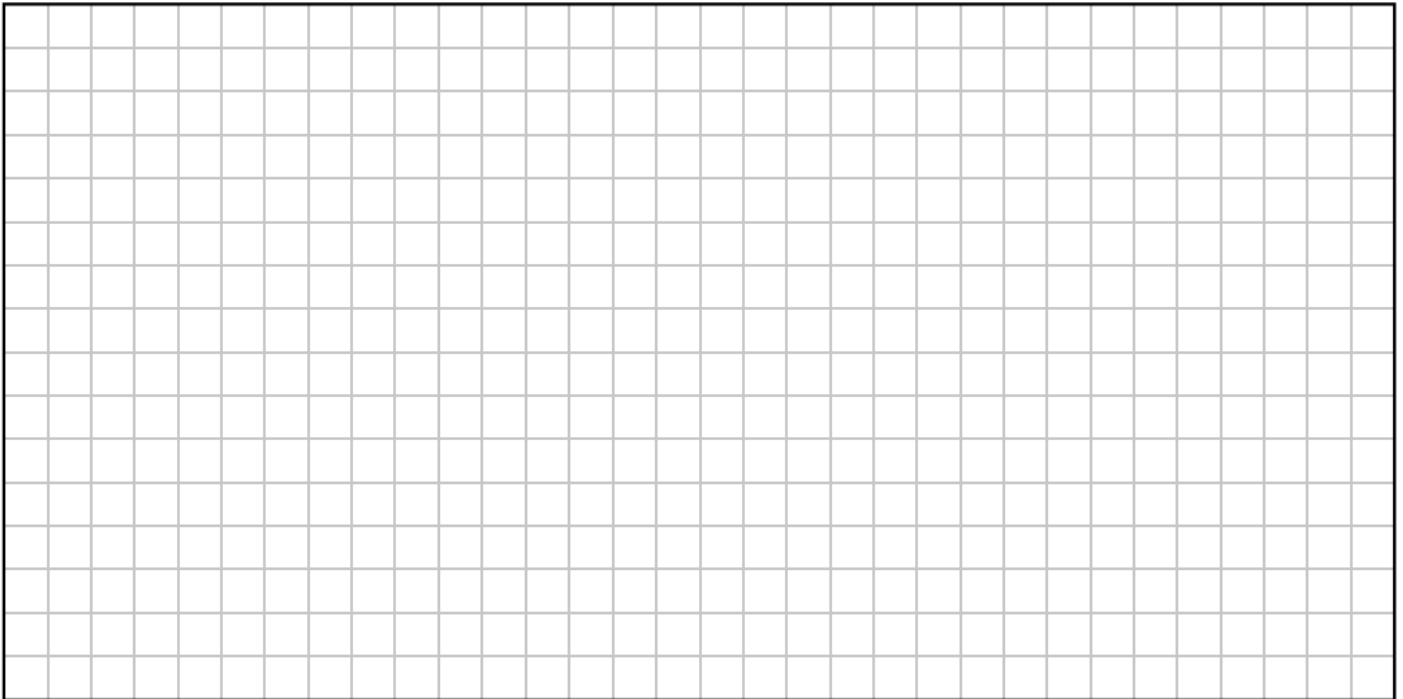
where v is her speed in km/hour t minutes after passing the point **A**, for $0 \leq t \leq 5$ and $t \in \mathbb{R}$.

- (a) Work out Fiona’s speed when she passes the point **A**.

- (b) Work out Fiona’s acceleration (that is, the rate at which her speed is increasing) 5 minutes after she passes the point **A**. Give your answer in km/hour per minute.

- (c) Find the time (value of t) at which Fiona reaches her maximum speed, during the first 4 minutes after she passes the point **A**. Give your answer correct to 2 decimal places.

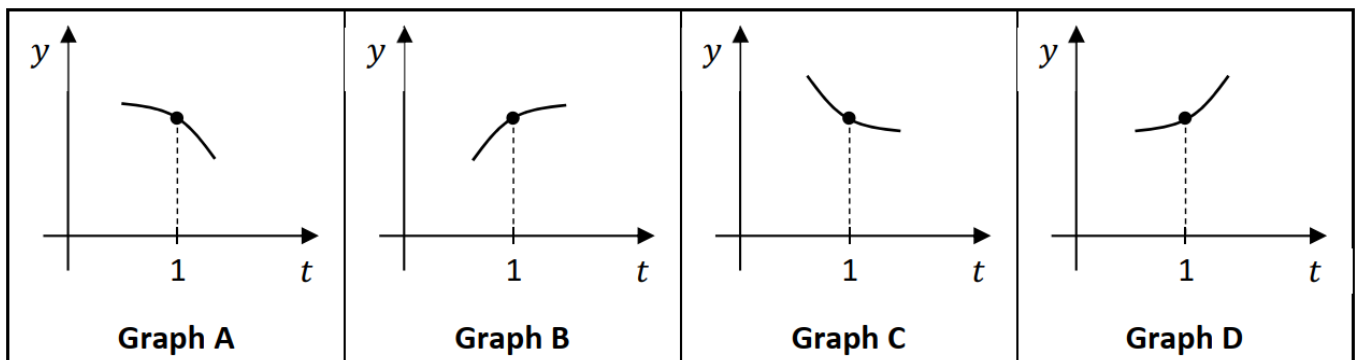
- (d) Use integration to work out Fiona's average speed over the 5 minutes after she passes the point A. Give your answer correct to 2 decimal places.



- (e) Taking $v'(t)$ to be the derivative of v , and $v''(t)$ to be the second derivative of v :

$$v'(1) > 0 \text{ and } v''(1) < 0.$$

Four graphs, A, B, C, and D, are shown below.



Close to where $t = 1$, the graph of $y = v(t)$ must look like one of the four graphs given above. Write down which graph this is. Justify your answer, using both $v'(1)$ and $v''(1)$.

Answer (A, B, C, or D):

Using $v'(1) > 0$:

Using $v''(1) < 0$:

This question continues on the next page.

There is an **Average Speed Zone** on the motorway, starting at the point **A** and ending at point **B**.
The distance from **A** to **B** along the motorway is 10 km.
Cameras record the time taken for each car to travel from the point **A** to the point **B**.
Each car's average speed from **A** to **B** is then calculated.

- (f) Work out the **minimum** time, in minutes, that a driver could get from **A** to **B**, while not driving above 100 km/hour.

- (g) Rohan drives from **A** to **B**.
He passes the point **A** driving at a constant speed of 120 km/hour. After 2 minutes driving at this speed, he starts to decelerate (reduce his speed) at a constant rate, until he reaches the point **B**.
Overall, his average speed in driving from **A** to **B** is 100 km/hour.

Work out Rohan's deceleration. Give your answer in km/hour per minute.