

► 2023 LCHL Paper 1 – Question 7

Fiona is driving on a motorway. She passes a point **A** on the motorway. Her speed is given by:

$$v(t) = \frac{2}{3}t^3 - 6t^2 + 13t + 109$$

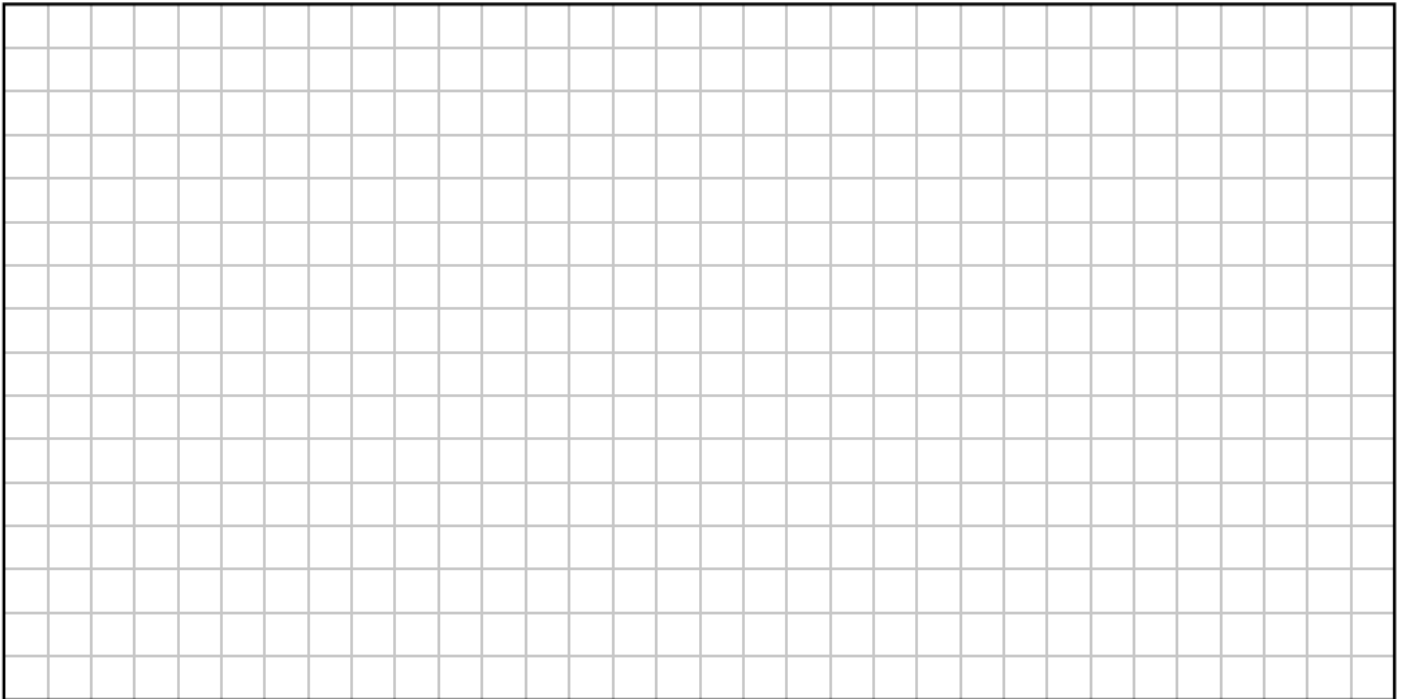
where  $v$  is her speed in km/hour  $t$  minutes after passing the point **A**, for  $0 \leq t \leq 5$  and  $t \in \mathbb{R}$ .

- (a) Work out Fiona's speed when she passes the point **A**.

- (b) Work out Fiona's acceleration (that is, the rate at which her speed is increasing) 5 minutes after she passes the point **A**. Give your answer in km/hour per minute.

- (c) Find the time (value of  $t$ ) at which Fiona reaches her maximum speed, during the first 4 minutes after she passes the point **A**. Give your answer correct to 2 decimal places.

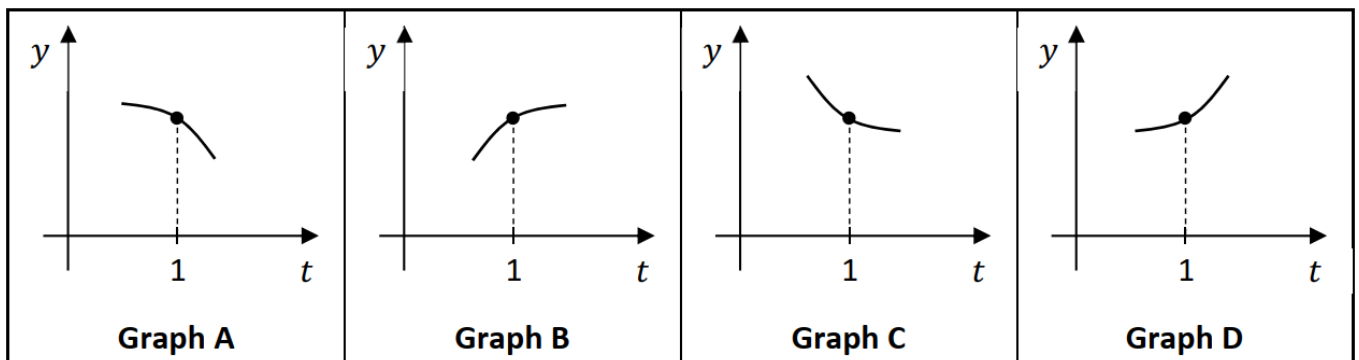
- (d) Use integration to work out Fiona's average speed over the 5 minutes after she passes the point A. Give your answer correct to 2 decimal places.



- (e) Taking  $v'(t)$  to be the derivative of  $v$ , and  $v''(t)$  to be the second derivative of  $v$ :

$$v'(1) > 0 \text{ and } v''(1) < 0.$$

Four graphs, A, B, C, and D, are shown below.



Close to where  $t = 1$ , the graph of  $y = v(t)$  must look like one of the four graphs given above. Write down which graph this is. Justify your answer, using both  $v'(1)$  and  $v''(1)$ .

Answer (A, B, C, or D):

Using  $v'(1) > 0$ :

Using  $v''(1) < 0$ :

*This question continues on the next page.*

There is an **Average Speed Zone** on the motorway, starting at the point **A** and ending at point **B**. The distance from **A** to **B** along the motorway is 10 km. Cameras record the time taken for each car to travel from the point **A** to the point **B**. Each car's average speed from **A** to **B** is then calculated.

- (f) Work out the **minimum** time, in minutes, that a driver could get from **A** to **B**, while not driving above 100 km/hour.

- (g) Rohan drives from **A** to **B**. He passes the point **A** driving at a constant speed of 120 km/hour. After 2 minutes driving at this speed, he starts to decelerate (reduce his speed) at a constant rate, until he reaches the point **B**. Overall, his average speed in driving from **A** to **B** is 100 km/hour.

Work out Rohan's deceleration. Give your answer in km/hour per minute.